Math 20
§1.6 Applications of Linear Systems

Example Part a: The economy of a small developing nation consists of three groups of producers. Farmers produce food, builders produce shelter, and tailors produce clothing.

Of the food produced by the farmers, $\frac{7}{16}$ is bought by the farmers themselves, $\frac{5}{16}$ by the builders, and $\frac{1}{4}$ by the tailors. Of the shelter produced by the builders, $\frac{1}{2}$ is bought by the farmers, $\frac{1}{6}$ by the builders themselves, and $\frac{1}{3}$ by the tailors. Of the clothing produced by the tailors, $\frac{3}{16}$ is bought by the farmers, $\frac{5}{16}$ by the builders, and $\frac{1}{2}$ by the tailors themselves.

Represent the above information in a matrix E, where the entry $e_{i,j}$ is the fraction of the total output of the jth industry purchased by the ith industry. (Label the farmers, builders, and tailors as industries 1, 2, and 3, respectively.) What do you notice about the columns of this matrix?

Part b: Let p_1 be the total value (in dollars) of all food produced by the farmers in a given year. (Thus p_1 dollars is the total income received by the farmers from food sales, and, for example, $\frac{5}{16}p_1$ dollars is the amount the builders spend on food.) Likewise, let p_2 be the total value (in dollars) of all shelter produced by the builders and p_3 be the total value (in dollars) of all clothing produced by the tailors.

Set up (but do not solve) a system of linear equations that answers the following question: What should p_1, p_2, and p_3 be so that this simple economy is in equilibrium, that is, so that the income of each group of producers is equal to the value of their expenses?

Part c: Solve this system of linear equations. What does the solution tell you about the equilibrium values of p_1, p_2, and p_3? Suppose we wanted the total value of all products to be 2200 dollars. What are the corresponding values of p_1, p_2, and p_3?