Math 20
§1.7 Linear Independence

Question 1: Which of the following is *not* a set of linearly dependent vectors?

1. $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
2. $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix}$
3. $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$
4. $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 6 \\ -12 \\ -21 \end{bmatrix}$

Question 2: Suppose that the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linear dependent.

True or False: One can write $\mathbf{v}_1$ as a linear combination of $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

1. True
2. False

Question 3: Suppose that the set of vectors $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent and that the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

True or False: One can write $\mathbf{v}_1$ as a linear combination of $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

1. True
2. False

Question 4: Suppose that the set of vectors $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent and that the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent. Which of the following statements must be true?

1. $\text{Span}\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = \text{Span}\{\mathbf{v}_3, \mathbf{v}_4\}$
2. $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = \text{Span}\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$
3. $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = \text{Span}\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$
4. $\text{Span}\{\mathbf{v}_1\} = \text{Span}\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

Question 5: Suppose that A is an $m \times n$ matrix with the property that for all $\mathbf{b}$ in $\mathbb{R}^m$ the equation $A\mathbf{x} = \mathbf{b}$ has at most one solution.

True or False: The columns of A must be linearly independent.

1. True
2. False

Question 6: Suppose that $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{bmatrix}$ is a 3×4 matrix with the properties that $\mathbf{a}_2$ is not a scalar multiple of $\mathbf{a}_1$ and $\mathbf{a}_3$ is not in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2\}$. Which of the following is a possible echelon form of A?

1. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$

2. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

3. $\begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

4. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$