Zero-sum problems

1. a)

$$\begin{array}{c|ccc} & R & P & S \\ \hline R & 0 & -1 & 1 \\ P & 1 & 0 & -1 \\ S & -1 & 1 & 0 \end{array} \qquad A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

b) $p = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} \qquad q = \begin{bmatrix} 0.75 \\ 0 \\ 0.25 \end{bmatrix}$

$$\text{Expected Payoff} = p^T A q = \begin{pmatrix} 0 & 0.5 & 0.5 \end{pmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.75 \\ 0 \\ 0.25 \end{bmatrix}$$

$$= -0.125$$

Column player wins more, since the expected payoff to him is positive.

2. $\begin{bmatrix} 2 & -2 & 0 \\ -6 & 0 & -5 \\ 5 & \boxed{2} & 3 \end{bmatrix}$ saddle point $= 2 \Rightarrow$ which gives Row player's optimal strategy as well as column player's optimal strategy

Value of the game $= 2$

3.

		C	
		Black 2	Red 3
R	Black Ace	3	−4
	Red 4	−6	7

$$p^* = \begin{bmatrix} \frac{7+6}{3+7+4+6} \\ \frac{3+4}{3+7+4+6} \end{bmatrix} = \begin{bmatrix} 0.65 \\ 0.35 \end{bmatrix}$$

$$q^* = \begin{bmatrix} \frac{7+4}{3+7+4+6} \\ \frac{3+6}{3+7+4+6} \end{bmatrix} = \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix}$$

$$\text{Value of the game} = \frac{(3)(7) - (-6)(-4)}{3+7+4+6} = -0.15$$