

Solution to A1 of Problem Set 4

We are given the initial conditions, $p_0 = 4$ in dollars, $x_0 = 40$ in thousands of dollars, $y_0 = 50$ in thousands of dollars and $z_0 = 12$ in hundreds of hours, and we are asked to approximate the value of S if we shift each of them by the small amount $dp = -1$, $dx = dy = dz = 1$. To solve this problem, we use

$$dS = \frac{\partial S}{\partial p} dp + \frac{\partial S}{\partial x} dx + \frac{\partial S}{\partial y} dy + \frac{\partial S}{\partial z} dz.$$

First we calculate all the partial derivatives:

$$\begin{aligned}\frac{\partial S}{\partial p} &= (-20)x^{1/5}y^{1/4}z^{1/2} \\ \frac{\partial S}{\partial x} &= (200 - 20p)\frac{1}{5x^{4/5}}y^{1/4}z^{1/2} \\ \frac{\partial S}{\partial y} &= (200 - 20p)x^{1/5}\frac{1}{4y^{3/4}}z^{1/2} \\ \frac{\partial S}{\partial z} &= (200 - 20p)x^{1/5}y^{1/4}\frac{1}{2z^{1/2}}\end{aligned}$$

Evaluating each at our point of interest, (4, 40, 50, 12):

$$\begin{aligned}S_p(4, 40, 50, 12) &= -385.28\$ \\ S_x(4, 40, 50, 12) &= 11.56\$ \\ S_y(4, 40, 50, 12) &= 11.56\$ \\ S_z(4, 40, 50, 12) &= 96.32\$\end{aligned}$$

Using our formula, we get

$$\begin{aligned}dS &= S_p(4, 40, 50, 12)dp + S_x(4, 40, 50, 12)dx + S_y(4, 40, 50, 12)dy + S_z(4, 40, 50, 12)dz \\ &= ((-385.28) * (-1) + 11.56 * 1 + 11.56 * 1 + 96.32 * 1)\$ \\ &= 504.72\$\end{aligned}$$

This number is positive and so the company will profit by this change.