

# Problem Set 5

Math 20

Due March 8, 2006

## Reading

Read Sections 11.6–11.8 in the Stewart excerpt.

## Book Problems

Please note that the problems given are numbered as in the Stewart excerpt, which is not the same as the third edition. Scans of the relevant pages from the excerpt have been posted on the course web site (under “Assignments”), so make sure you are doing the right problems. *No credit will be given for doing the wrong problems.*

- 11.6: 8, 10, 20, 30
- 11.7: 2, 4, 6, 10, 45
- 11.8: 4, 12, 18

## Additional Problems

B1. Let  $\alpha$ ,  $\beta$ ,  $p$ ,  $q$ , and  $b$  be positive constants. Find the maximum point and maximum value of

$$u\left(\begin{matrix} x \\ y \end{matrix}\right) = x^\alpha y^\beta$$

subject to the constraints  $x \geq 0$ ,  $y \geq 0$ ,  $px + qy \leq b$ .

B2. (Continued) Suppose the above model is used for the familiar problem of maximizing utility subject to budget constraints. Given  $b$ , we find  $x$  and  $y$  to maximize  $u$ . This gives a function  $u(b)$ . Show that

$$\frac{du}{db} = \lambda,$$

in other words, *the Lagrange multiplier is the marginal utility of money.*

B3. (Continued)

- (a) Suppose the government levies an income tax, taking  $\rho$  dollars out of every dollar earned. How does the consumer's utility change?
- (b) Suppose instead the government levies a sales tax on the first good, charging  $\tau$  dollars for every dollar spent on it. How does the consumer's utility change?
- (c) Would the consumer rather have a sales tax or an income tax if both were to generate the same amount for the government?
- B4. At regular intervals, a firm orders a quantity  $x$  of a commodity which is placed in stock. This stock is depleted at a constant rate until none remains whereupon the firm immediately restocks with quantity  $x$ .

The firm requires  $X$  units of the commodity each year and on average, the firm orders the commodity with a frequency of  $y$  times in each year. If the requirement is to be met, we must have  $xy = X$ .

Let the cost of holding one unit of the commodity in stock for a year be  $d$ . Since the average amount of the commodity held in stock is  $\frac{1}{2}x$ , it follows that the yearly holding cost is  $\frac{1}{2}xd$ . Let the cost of reordering the commodity be  $e$ . The yearly reordering cost is therefore  $ey$ .

The problem that the firm (and you, their chief consultant) faces is to minimize cost

$$C \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}xd + ey$$

subject to the constraint  $xy = X$ .

## Division

- Part I: Section 11.6 and 11.7
- Part II: Section 11.8 and exercises B