10.

12.

14. $x = \frac{18}{5}$, $y = \frac{2}{5}$, optimal value of z is $\frac{58}{5}$.

16. $\frac{3}{2}$ tons of regular steel and $\frac{5}{2}$ tons of special steel; maximum profit is \$430.

18. Invest \$4000 in bond A and \$2000 in bond B; maximum return is \$520.

20. Use 2 minutes of advertising and 28 minutes of programming; maximum number of viewer-minutes is 1,340,000.

22. Use 4 units of A and 3 units of B; maximum amount of protein is 34 units.

24. (b).

26. Maximize $z = 2x_1 - 3x_2 - 2x_3$

subject to

$$
\begin{aligned}
2x_1 + x_2 + 2x_3 &\le 12 \\
x_1 + x_2 - 3x_3 &\le 8 \\
x_1 \ge 0, x_2 \ge 0, x_3 &\ge 0
\end{aligned}
$$

28. Maximize $z = 2x + 8y$

subject to

$$
\begin{aligned}
2x + 3y + u \quad\; &= \quad 18 \\
3x - 2y \quad\; + v &= \quad 6 \\
x \ge 0, y \ge 0, u \ge 0, v &\ge 0
\end{aligned}
$$

Section 11.2, p. 589

2.

	x_1	x_2	x_3	x_4	x_5	z	
x_4	3	-2	1	1	0	0	4
x_5	2	4	5	0	1	0	6
	-2	-3	4	0	0	1	0

4.

	x_1	x_2	x_3	x_4	x_5	x_6	z	
x_4	1	-2	4	1	0	0	0	5
x_5	2	2	4	0	1	0	0	5
x_6	3	1	-1	0	0	1	0	7
	-2	3	-1	0	0	0	1	0

6. $x = 0$, $y = \frac{6}{7}$, optimal z value is $\frac{30}{7}$.

8. No finite optimal solution.

10. $x_1 = 0$, $x_2 = \frac{5}{2}$, $x_3 = 0$, optimal z value is $z = 10$.

12. $\frac{3}{2}$ tons of regular steel and $\frac{5}{2}$ tons of special steel; maximum profit is \$430.

14. 4 units of A and 3 units of B; maximum amount of protein is 34 units.

T.1. We must show that if $\mathbf{x}$ and $\mathbf{y}$ are any two feasible solutions, then for any $0 \le r \le 1$, the vector $r\mathbf{x} + (1 - r)\mathbf{y}$ is also a feasible solution. First, since $r \ge 0$ and $(1 - r) \ge 0$, and $A\mathbf{x} \le \mathbf{b}$, $A\mathbf{y} \le \mathbf{b}$,

$$A[r\mathbf{x} + (1 - r)\mathbf{y})] = rA\mathbf{x} + (1 - r)A\mathbf{y} \le r\mathbf{b} + (1 - r)\mathbf{b} = \mathbf{b}.$$

Also, since $\mathbf{x} \ge \mathbf{0}$, $\mathbf{y} \ge \mathbf{0}$,

$$r\mathbf{x} + (1 - r)\mathbf{y} \ge r \cdot \mathbf{0} + (1 - r) \cdot \mathbf{0} = \mathbf{0}.$$

Thus $r\mathbf{x} + (1 - r)\mathbf{y}$ is a feasible solution.

T.2. Suppose that in a certain step of the simplex method, the minimum positive θ-ratio is not chosen. After a reindexing of the variables, if necessary, we may assume that all the nonbasic variables occur first followed by all the basic variables. That is, we may assume that we start with the situation given by Tableau 2 which precedes Equation (16). Let us further assume that x_1 is the entering variable and x_{n+1} the departing variable and that the θ-ratio b_2/a_{21} associated with the second row is positive and smaller than b_1/a_{11} associated with the first row — the row of the incorrectly chosen departing variable x_{n+1}. Thus

$$0 < \frac{b_2}{a_{21}} < \frac{b_1}{a_{11}}$$
$$a_{11}b_2 < a_{21}b_1,$$
$$a_{11}b_2 - a_{21}b_1 < 0.$$

The new set of nonbasic variables is $\{x_2, \ldots, x_n, x_{n+1}\}$. Set these nonbasic variables equal to zero and solve the first equation for x_1:

$$a_{11}x_1 + a_{12} \cdot 0 + \cdots + a_{1n} \cdot 0 + 0 = b_1$$
$$x_1 = \frac{b_1}{a_{11}}.$$

Next substitute this value for x_1 into the second equation and solve for the basic variable x_{n+2}:

$$a_{21}\left(\frac{b_1}{a_{11}}\right) + a_{22} \cdot 0 + \cdots + a_{2n} \cdot 0 + x_{n+2} = b_2,$$
$$x_{n+2} = b_2 - \frac{a_{21}b_1}{a_{11}} = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}} = \frac{\text{neg}}{\text{pos}} = \text{neg},$$

a contradiction to the fact that the coordinate x_{n+2} of any feasible solution must be ≥ 0.

ML.5. As a check, the solution is $x_4 = \frac{2}{3}$, $x_3 = 0$, $x_2 = \frac{1}{3}$, $x_1 = 1$, all other variables zero and the optimal value of z is 11. The final tableau is as follows:

0	0	1.6667	1	0.0000	−0.1667	0	0.6667
0	1	3.3333	0	1.0000	−0.3333	0	0.3333
1	0	−3.0000	0	−1.0000	0.5000	0	1.0000
0	0	1.0000	0	1.0000	1.0000	1	11.0000

ML.6. As a check, the solution is $x_3 = \frac{4}{3}$, $x_2 = 4$, $x_7 = \frac{22}{3}$, all other variables zero, and the optimal value of z is $\frac{28}{3}$. The final tableau is as follows:

0.4444	0	1	−0.1111	0.3333	−0.1111	0	0	1.3333
0.6667	1	0	1.3333	0	0.3333	0	0	4.0000
0.7778	0	0	−2.4444	−0.6667	−0.4444	1	0	7.3333
0.7778	0	0	1.5556	0.3333	0.5556	0	1	9.3333

Section 11.3, p. 598

2. Maximize $z' = 5w_1 + 6w_2$

   subject to

   $$w_1 + 2w_2 \geq 10$$
   $$3w_1 - 4w_2 \geq 12$$
   $$4w_1 - 5w_2 \geq 15$$
   $$w_1 \geq 0, w_2 \geq 0$$

4. Maximize $z' = 9w_1 + 12w_2$

   subject to

   $$3w_1 + 5w_2 \leq 14$$
   $$5w_1 + 2w_2 \leq 12$$
   $$-4w_1 + 7w_2 \leq 18$$
   $$w_1 \geq 0, w_2 \geq 0$$

6. $w_1 = \frac{2}{3}$, $w_2 = 0$, optimal value is 4.

8. $w_1 = \frac{1}{10}$, $w_2 = \frac{7}{10}$, optimal value is $\frac{69}{10}$.

10. Use 6 oz. of dates and no nuts or raisins. Total cost is 90 cents.

Section 11.4, p. 612

2.

		C		
		stone	scissors	paper
	stone	0	1	−1
R	scissors	−1	0	1
	paper	1	−1	0