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3. If the amounts of labor and capital are both doubled, we replace L, K in the function with $2L, 2K$, giving

$$P(2L, 2K) = 1.01(2L)^{0.75}(2K)^{0.25} = 1.01(2^{0.75})(2^{0.25})L^{0.75}K^{0.25} = (2^1)1.01L^{0.75}K^{0.25} = 2P(L, K)$$

Thus, the production is doubled. It is also true for the general case $P(L, K) = bL^{\alpha}K^{1-\alpha}$:

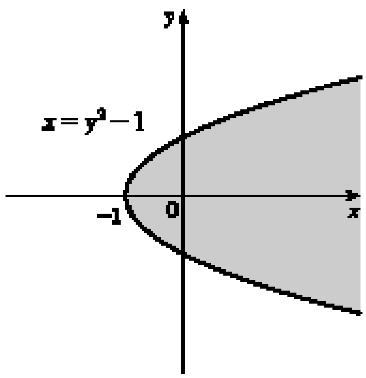
$P(2L, 2K) = b(2L)^{\alpha}(2K)^{1-\alpha} = b(2^{\alpha})(2^{1-\alpha})L^{\alpha}K^{1-\alpha} = (2^{\alpha+1-\alpha})bL^{\alpha}K^{1-\alpha} = 2P(L, K)$.

6. $\sqrt{1+x-y^2}$ is defined only when $1+x-y^2 \geq 0 \quad \Rightarrow$

$x \geq y^2 - 1$, so the domain of f is $\{(x, y) \mid x \geq y^2 - 1\}$,

all those points on or to the right of the parabola $x = y^2 - 1$.

The range of f is $[0, \infty)$.



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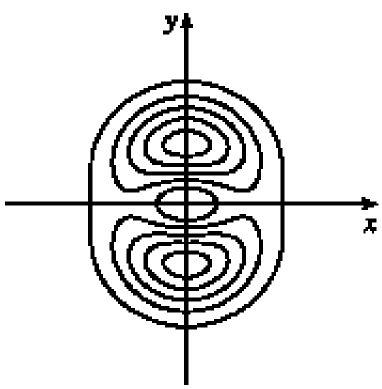

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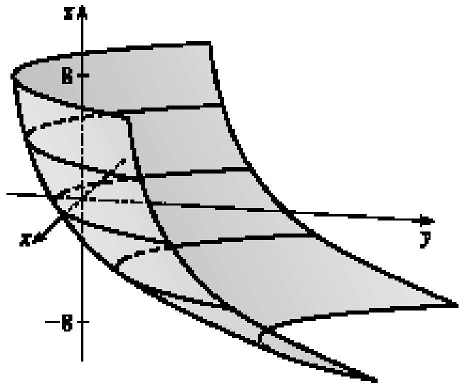
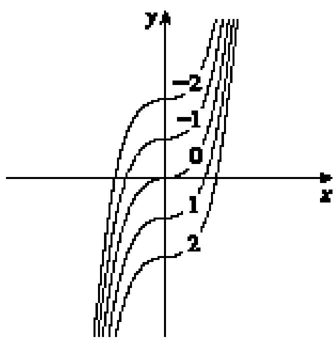
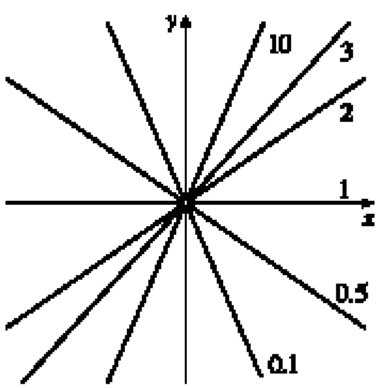


16. The level curves are $x^3 - y = k$ or $y = x^3 - k$, a family of cubic curves.



18. The level curves are $e^{y/x} = k$ or equivalently $y = x \ln k$ ($x \neq 0$), a family of lines with slope $\ln k$ ($k > 0$) without the origin.



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2. (a) The outdoor temperature as a function of longitude, latitude, and time is continuous. Small changes in longitude, latitude, or time can produce only small changes in temperature, as the temperature doesn't jump abruptly from one value to another.

(b) Elevation is not necessarily continuous. If we think of a cliff with a sudden drop-off, a very small change in longitude or latitude can produce a comparatively large change in elevation, without all the intermediate values being attained. Elevation *can* jump from one value to another.

(c) The cost of a taxi ride is usually discontinuous. The cost normally increases in jumps, so small changes in distance traveled or time can produce a jump in cost. A graph of the function would show breaks in the surface.

4. We make a table of values of $f(x,y) = \dfrac{2xy}{x^2+2y^2}$ for a set of (x,y) points near the origin.

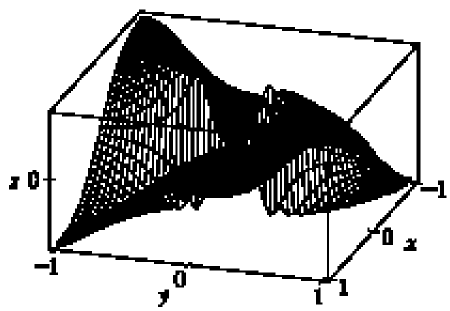
$x \backslash y$	−0.3	−0.2	−0.1	0	0.1	0.2	0.3
−0.3	0.667	0.706	0.545	0.000	−0.545	−0.706	−0.667
−0.2	0.545	0.667	0.667	0.000	−0.667	−0.667	−0.545
−0.1	0.316	0.444	0.667	0.000	−0.667	−0.444	−0.316
0	0.000	0.000	0.000		0.000	0.000	0.000
0.1	−0.316	−0.444	−0.667	0.000	0.667	0.444	0.316
0.2	−0.545	−0.667	−0.667	0.000	0.667	0.667	0.545
0.3	−0.667	−0.706	−0.545	0.000	0.545	0.706	0.667

It appears from the table that the values of $f(x,y)$ are not approaching a single value as (x,y) approaches the origin. For verification, if we first approach $(0,0)$ along the x-axis, we have $f(x,0)=0$, so $f(x,y)\to 0$. But if we approach $(0,0)$ along the line $y=x$, $f(x,x) = \dfrac{2x^2}{x^2+2x^2} = \dfrac{2}{3}$ $(x \neq 0)$, so $f(x,y) \to \frac{2}{3}$. Since f approaches different values along different paths to the origin, this limit does not exist.

6. $x-2y$ is a polynomial and therefore continuous. Since $\cos t$ is a continuous function, the composition $\cos(x-2y)$ is also continuous. xy is also a polynomial, and hence continuous, so the product $f(x,y) = xy\cos(x-2y)$ is a continuous function. Then $\lim\limits_{(x,y)\to(6,3)} f(x,y) = f(6,3) = (6)(3)\cos(6-2\cdot 3) = 18$.

8. $f(x,y) = (x^2 + \sin^2 y)/(2x^2 + y^2)$. First approach $(0,0)$ along the x-axis. Then $f(x,0) = x^2/2x^2 = \frac{1}{2}$ for $x \neq 0$, so $f(x,y) \to \frac{1}{2}$. Next approach $(0,0)$ along the y-axis. For $y \neq 0$, $f(0,y) = \dfrac{\sin^2 y}{y^2} = \left(\dfrac{\sin y}{y}\right)^2$ and $\lim\limits_{y \to 0} \dfrac{\sin y}{y} = 1$, so $f(x,y) \to 1$. Since f has two different limits along two different lines, the limit does not exist.

20.



From the graph, it appears that as we approach the origin along the lines $x = 0$ or $y = 0$, the function is everywhere 0, whereas if we approach the origin along a certain curve it has a constant value of about $\frac{1}{2}$. [In fact, $f(y^3, y) = y^6/(2y^6) = \frac{1}{2}$ for $y \neq 0$, so $f(x,y) \to \frac{1}{2}$ as $(x,y) \to (0,0)$ along the curve $x = y^3$.] Since the function approaches different values depending on the path of approach, the limit does not exist.

6. (a) The graph of f decreases if we start at $(-1,2)$ and move in the positive x-direction, so $f_x(-1,2)$ is negative.

(b) The graph of f decreases if we start at $(-1,2)$ and move in the positive y-direction, so $f_y(-1,2)$ is negative.

(c) $f_{xx} = \frac{\partial}{\partial x}(f_x)$, so f_{xx} is the rate of change of f_x in the x-direction. f_x is negative at $(-1,2)$ and if we move in the positive x-direction, the surface becomes less steep. Thus the values of f_x are increasing and $f_{xx}(-1,2)$ is positive.

(d) f_{yy} is the rate of change of f_y in the y-direction. f_y is negative at $(-1,2)$ and if we move in the positive y-direction, the surface becomes steeper. Thus the values of f_y are decreasing, and $f_{yy}(-1,2)$ is negative.

8. $f_x(2,1)$ is the rate of change of f at $(2,1)$ in the x-direction. If we start at $(2,1)$, where $f(2,1) = 10$, and move in the positive x-direction, we reach the next contour line (where $f(x,y) = 12$) after approximately 0.6 units. This represents an average rate of change of about $\frac{2}{0.6}$. If we approach the point $(2,1)$ from the left (moving in the positive x-direction) the output values increase from 8 to 10 with an increase in x of approximately 0.9 units, corresponding to an average rate of change of $\frac{2}{0.9}$. A good estimate for $f_x(2,1)$ would be the average of these two, so $f_x(2,1) \approx 2.8$. Similarly, $f_y(2,1)$ is the rate of change of f at $(2,1)$ in the y-direction. If we approach $(2,1)$ from below, the output values decrease from 12 to 10 with a change in y of approximately 1 unit, corresponding to an average rate of change of -2. If we start at $(2,1)$ and move in the positive y-direction, the output values decrease from 10 to 8 after approximately 0.9 units, a rate of change of $\frac{-2}{0.9}$. Averaging these two results, we estimate $f_y(2,1) \approx -2.1$.

14. $f(x,y) = x^5 + 3x^3y^2 + 3xy^4 \quad \Rightarrow \quad f_x(x,y) = 5x^4 + 3 \cdot 3x^2 \cdot y^2 + 3 \cdot 1 \cdot y^4 = 5x^4 + 9x^2y^2 + 3y^4$,
$f_y(x,y) = 0 + 3x^3 \cdot 2y + 3x \cdot 4y^3 = 6x^3y + 12xy^3$.

20. $f(s,t) = \dfrac{st^2}{s^2+t^2} \quad \Rightarrow$

$$f_s(s,t) = \frac{t^2(s^2+t^2) - st^2(2s)}{(s^2+t^2)^2} = \frac{t^4 - s^2t^2}{(s^2+t^2)^2}, \quad f_t(s,t) = \frac{2st(s^2+t^2) - st^2(2t)}{(s^2+t^2)^2} = \frac{2s^3t}{(s^2+t^2)^2}$$

26. $f(x,y,z) = x^2e^{yz} \quad \Rightarrow \quad f_x(x,y,z) = 2xe^{yz}$, $f_y(x,y,z) = x^2e^{yz}(z) = x^2ze^{yz}$, $f_z(x,y,z) = x^2e^{yz}(y) = x^2ye^{yz}$

38. $f(u,v,w) = w\tan(uv) \quad \Rightarrow \quad f_v(u,v,w) = w\sec^2(uv) \cdot u = uw\sec^2(uv)$, so $f_v(2,0,3) = (2)(3)\sec^2(2 \cdot 0) = 6$.

42. $yz = \ln(x+z) \quad \Rightarrow \quad \dfrac{\partial}{\partial x}(yz) = \dfrac{\partial}{\partial x}(\ln(x+z)) \quad \Rightarrow \quad y\dfrac{\partial z}{\partial x} = \dfrac{1}{x+z}\left(1 + \dfrac{\partial z}{\partial x}\right) \quad \Leftrightarrow \quad \left(y - \dfrac{1}{x+z}\right)\dfrac{\partial z}{\partial x} = \dfrac{1}{x+z}$,

so $\dfrac{\partial z}{\partial x} = \dfrac{1/(x+z)}{y - 1/(x+z)} = \dfrac{1}{y(x+z) - 1}$.

$\dfrac{\partial}{\partial y}(yz) = \dfrac{\partial}{\partial y}(\ln(x+z)) \quad \Rightarrow \quad y\dfrac{\partial z}{\partial y} + z \cdot 1 = \dfrac{1}{x+z}\left(0 + \dfrac{\partial z}{\partial y}\right) \quad \Leftrightarrow \quad \left(y - \dfrac{1}{x+z}\right)\dfrac{\partial z}{\partial y} = -z$,

so $\dfrac{\partial z}{\partial y} = \dfrac{-z}{y - 1/(x+z)} = \dfrac{z(x+z)}{1 - y(x+z)}$.

46. (a) $z = f(x)g(y) \quad \Rightarrow \quad \dfrac{\partial z}{\partial x} = f'(x)g(y)$, $\dfrac{\partial z}{\partial y} = f(x)g'(y)$

(b) $z = f(xy)$. Let $u = xy$. Then $\dfrac{\partial u}{\partial x} = y$ and $\dfrac{\partial u}{\partial y} = x$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du} \cdot y = yf'(u) = yf'(xy)$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du} \cdot x = xf'(u) = xf'(xy)$.

(c) $z = f\left(\dfrac{x}{y}\right)$. Let $u = \dfrac{x}{y}$. Then $\dfrac{\partial u}{\partial x} = \dfrac{1}{y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{x}{y^2}$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = f'(u)\dfrac{1}{y} = \dfrac{f'(x/y)}{y}$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = f'(u)\left(-\dfrac{x}{y^2}\right) = -\dfrac{xf'(x/y)}{y^2}$.

48. $f(x,y) = \ln(3x+5y) \quad \Rightarrow \quad f_x(x,y) = \dfrac{3}{3x+5y}$, $f_y(x,y) = \dfrac{5}{3x+5y}$. Then

$f_{xx}(x,y) = 3(-1)(3x+5y)^{-2}(3) = -\dfrac{9}{(3x+5y)^2}$, $f_{xy}(x,y) = -\dfrac{15}{(3x+5y)^2}$, $f_{yx}(x,y) = -\dfrac{15}{(3x+5y)^2}$,

and $f_{yy}(x,y) = -\dfrac{25}{(3x+5y)^2}$.

50. $z = y\tan 2x \quad \Rightarrow \quad z_x = y\sec^2(2x)\cdot 2 = 2y\sec^2(2x)$, $z_y = \tan 2x$. Then

$z_{xx} = 2y(2)\sec(2x)\cdot\sec(2x)\tan(2x)\cdot 2 = 8y\sec^2(2x)\tan(2x)$, $z_{xy} = 2\sec^2(2x)$, $z_{yx} = \sec^2(2x)\cdot 2 = 2\sec^2(2x)$,

and $z_{yy} = 0$.