Name: ___________________ Class:___ Date: _____________

2. $z = f(x, y) = e^{x^2 - y^2} \quad \Rightarrow \quad f_x(x, y) = 2xe^{x^2 - y^2}$, $f_y(x, y) = -2ye^{x^2 - y^2}$, so $f_x(1, -1) = 2$, $f_y(1, -1) = 2$. By Equation 2, an equation of the tangent plane is $z - 1 = f_x(1, -1)(x - 1) + f_y(1, -1)[y - (-1)] \quad \Rightarrow$
$z - 1 = 2(x - 1) + 2(y + 1)$ or $z = 2x + 2y + 1$.

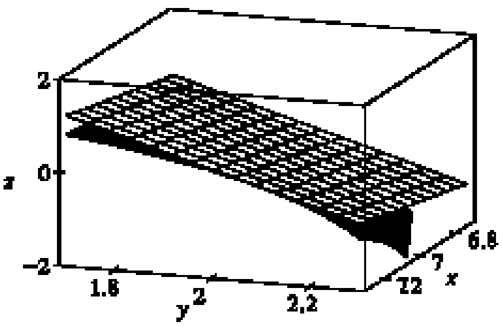
4. $z = f(x, y) = y \ln x \quad \Rightarrow \quad f_x(x, y) = y/x$, $f_y(x, y) = \ln x$, so $f_x(1, 4) = 4$, $f_y(1, 4) = 0$, and an equation of the tangent plane is $z - 0 = f_x(1, 4)(x - 1) + f_y(1, 4)(y - 4) \quad \Rightarrow \quad z = 4(x - 1) + 0(y - 4)$ or $z = 4x - 4$.

14. $f(x, y) = \ln(x - 3y) \quad \Rightarrow \quad f_x(x, y) = \dfrac{1}{x - 3y}$ and $f_y(x, y) = -\dfrac{3}{x - 3y}$, so $f_x(7, 2) = 1$ and $f_y(7, 2) = -3$.

Then the linear approximation of f at $(7, 2)$ is given by

$$\begin{aligned} f(x, y) &\approx f(7, 2) + f_x(7, 2)(x - 7) + f_y(7, 2)(y - 2) \\ &= 0 + 1(x - 7) - 3(y - 2) = x - 3y - 1 \end{aligned}$$

Thus $f(6.9, 2.06) \approx 6.9 - 3(2.06) - 1 = -0.28$. The graph shows that our approximated value is slightly greater than the actual value.



20. $u = e^{-t} \sin(s + 2t) \quad \Rightarrow$

$$\begin{aligned} du &= \frac{\partial u}{\partial s}\,ds + \frac{\partial u}{\partial t}\,dt = e^{-t}\cos(s + 2t)\,ds + \left[e^{-t}\cos(s + 2t) \cdot 2 + \left(-e^{-t}\right)\sin(s + 2t)\right] dt \\ &= e^{-t}\cos(s + 2t)\,ds + e^{-t}\left[2\cos(s + 2t) - \sin(s + 2t)\right] dt \end{aligned}$$

22. $w = xye^{xz} \quad \Rightarrow$

$$\begin{aligned} dw &= \frac{\partial w}{\partial x}\,dx + \frac{\partial w}{\partial y}\,dy + \frac{\partial w}{\partial z}\,dz = \left(xyze^{xz} + ye^{xz}\right)dx + xe^{xz}\,dy + x^2ye^{xz}\,dz \\ &= (xz + 1)ye^{xz}\,dx + xe^{xz}\,dy + x^2ye^{xz}\,dz. \end{aligned}$$

4. $w = xy + yz^2$, $x = e^t$, $y = e^t \sin t$, $z = e^t \cos t \quad \Rightarrow$

$$\begin{aligned} \frac{dw}{dt} &= \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} = y \cdot e^t + (x + z^2) \cdot (e^t \cos t + e^t \sin t) + 2yz \cdot (-e^t \sin t + e^t \cos t) \\ &= e^t\left[y + (x + z^2)(\cos t + \sin t) + 2yz(\cos t - \sin t)\right] \end{aligned}$$

6. $z = x/y,\ x = se^t,\ y = 1 + se^{-t} \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \frac{1}{y}(e^t) + \left(-\frac{x}{y^2}\right)(e^{-t}) = \frac{1}{y}\,e^t - \frac{x}{y^2}\,e^{-t}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \frac{1}{y}(se^t) + \left(-\frac{x}{y^2}\right)(-se^{-t}) = \frac{s}{y}\,e^t + \frac{xs}{y^2}\,e^{-t}$$

10. By the Chain Rule (3), $\dfrac{\partial W}{\partial s} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial s} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial s}$. Then

$$\begin{aligned} W_s(1,0) &= F_u(u(1,0), v(1,0))\,u_s(1,0) + F_v(u(1,0), v(1,0))\,v_s(1,0) \\ &= F_u(2,3)u_s(1,0) + F_v(2,3)v_s(1,0) = (-1)(-2) + (10)(5) = 52 \end{aligned}$$

Similarly, $\dfrac{\partial W}{\partial t} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial t} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial t} \quad \Rightarrow$

$$\begin{aligned} W_t(1,0) &= F_u(u(1,0), v(1,0))\,u_t(1,0) + F_v(u(1,0), v(1,0))\,v_t(1,0) \\ &= F_u(2,3)u_t(1,0) + F_v(2,3)v_t(1,0) = (-1)(6) + (10)(4) = 34 \end{aligned}$$

12. $g(r,s) = f(x(r,s), y(r,s))$ where $x = 2r - s,\ y = s^2 - 4r \quad \Rightarrow \quad \dfrac{\partial x}{\partial r} = 2,\ \dfrac{\partial x}{\partial s} = -1,\ \dfrac{\partial y}{\partial r} = -4,\ \dfrac{\partial y}{\partial s} = 2s$. By the Chain Rule (3) $\dfrac{\partial g}{\partial r} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial r}$. Then

$$\begin{aligned} g_r(1,2) &= f_x(x(1,2), y(1,2))\,x_r(1,2) + f_y(x(1,2), y(1,2))\,y_r(1,2) \\ &= f_x(0,0)(2) + f_y(0,0)(-4) = 4(2) + 8(-4) = -24 \end{aligned}$$

Similarly $\dfrac{\partial g}{\partial s} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial s}$. Then

$$\begin{aligned} g_s(1,2) &= f_x(x(1,2), y(1,2))\,x_s(1,2) + f_y(x(1,2), y(1,2))\,y_s(1,2) \\ &= f_x(0,0)(-1) + f_y(0,0)(4) = 4(-1) + 8(4) = 28 \end{aligned}$$

28. $yz = \ln(x+z)$, so let $F(x,y,z) = yz - \ln(x+z) = 0$. Then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z} = -\dfrac{-\dfrac{1}{x+z}\,(1)}{y - \dfrac{1}{x+z}\,(1)} = \dfrac{1}{y\,(x+z) - 1}$,

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{z}{y - \dfrac{1}{x+z}} = -\frac{z\,(x+z)}{y\,(x+z) - 1}.$$

29. Since x and y are each functions of t, $T(x,y)$ is a function of t, so by the Chain Rule, $\frac{dT}{dt} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt}$. After 3 seconds, $x = \sqrt{1+t} = \sqrt{1+3} = 2$, $y = 2 + \frac{1}{3}t = 2 + \frac{1}{3}(3) = 3$, $\frac{dx}{dt} = \frac{1}{2\sqrt{1+t}} = \frac{1}{2\sqrt{1+3}} = \frac{1}{4}$, and $\frac{dy}{dt} = \frac{1}{3}$. Then $\frac{dT}{dt} = T_x(2,3)\,\frac{dx}{dt} + T_y(2,3)\,\frac{dy}{dt} = 4\left(\frac{1}{4}\right) + 3\left(\frac{1}{3}\right) = 2$. Thus the temperature is rising at a rate of $2^\circ\mathrm{C/s}$.