

Midterm 1

1. Solve the following system of linear equations.

$$\begin{array}{rcccccc} x_1 & - & x_2 & - & 5x_3 & = & -1 \\ -2x_1 & + & 2x_2 & + & 11x_3 & = & 1 \\ 3x_1 & - & x_2 & + & x_3 & = & 3 \end{array}$$

2. Given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, find $(3AB)^T - B^T A^T$.

3. Given that $(I - 3A)^{-1} = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$, find the matrix A .

4. Given that $A = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 2 & 10 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}$, find an elementary matrix E such that $EA = B$.

5. Given that $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, find A^{-1} .

6. Given that $A = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$ and $\det(A) = 4$, evaluate the following determinants.

$$(a) \begin{vmatrix} 0 & 0 & 3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix}$$

$$(b) \begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 5 & -6 & 7 \\ 0 & 0 & -8 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$(c) \begin{vmatrix} x & y & z \\ u & v & w \\ r & s & t \end{vmatrix}$$

$$(d) \begin{vmatrix} r & s & t \\ 8u & 8v & 8w \\ x - 8r & y - 8s & z - 8t \end{vmatrix}$$

(e) $\det(A^{-1})$

(f) $\det(A^2)$

7. Evaluate the determinant $\begin{vmatrix} 1 & 7 & 2 & 0 \\ 2 & 5 & 5 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix}$.

8. Let $A = \begin{bmatrix} 3 & 0 \\ 1 & 9 \end{bmatrix}$.

(a) Find the eigenvalues of the matrix A .

(b) Select one of the eigenvalues you found in part (a) and find its corresponding eigenvector.

9. Explain why the following statement is true. “If $\det(A) = 0$, then the homogeneous system $Ax = 0$ has infinitely many solutions.”

10. In each round of the two-person game *rock-paper-scissors*, each player chooses one of rock, paper, or scissors. Rock beats scissors, scissors beats paper, and paper beats rock. If both players choose the same object, then the round is considered a tie.

(a) Give the payoff matrix for this game, assuming that an entry of 1 indicates a win for the row player, an entry of -1 indicates a loss for the row player, and an entry of 0 indicates a tie. Label the rows and columns of the matrix with the row and column player actions.

(b) If the row player chooses paper 50% of the time and scissors 50% of the time and if the column player chooses rock 75% of the time and scissors 25% of the time, which player will win more rounds of the game on average?