

Name: \_\_\_\_\_ ID#: \_\_\_\_\_

# Final Exam

Math 20  
Introduction to Linear Algebra  
and Multivariable Calculus

January 21, 2005

**Show all of your work. Full credit may not be given for an answer alone.** You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

**This is a non-calculator exam.**

*Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.*

*—Handbook for Students*

Problem Number	Possible Points	Points Earned
1	10	
2	10	
3	15	
4	15	
5	10	
6	10	
7	20	
8	10	
9	10	
10	10	
Total	120	

## Part I. Techniques

**1****1**

1. (10 Points) Find the Reduced Row Echelon Form of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix}.$$

**In this and in any problem where you do Gaussian Elimination, label your row operations to receive full or partial credit.**

**2**

**2**

2. (10 Points) Find the parametric form of the solution to the system of linear equations

$$\begin{aligned}x_1 + 2x_2 + 3x_3 + x_4 &= 8 \\x_1 + 3x_2 + x_4 &= 7 \\x_1 + 2x_2 + x_4 &= 3\end{aligned}$$

**3**

**3**

3. (15 Points) Let

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix}.$$

Find the inverse of  $A$ .

**4**

**4**

4. (15 Points)  $A$  is the same matrix as before:

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix}.$$

Find a diagonal matrix  $D$  and an invertible matrix  $P$  such that  $A = PDP^{-1}$ .

**5**

**5**

5. (10 Points) Let

$$f(x, y) = xy.$$

- (i) Find and classify all critical points of the function.

5

5

(ii) Find the maximum and minimum value of  $f$  on the ellipse

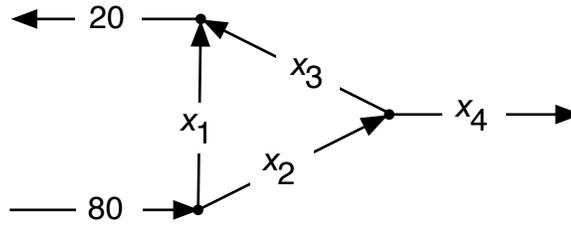
$$x^2 + \frac{y^2}{4} = 1.$$

## Part II. Applications

6

6

6. (10 Points) Consider the network with flow given as below:



Find the maximum value of  $x_3$ , assuming all flow rates are nonnegative.

7. (20 Points) A pension fund has \$30 million to invest. The money is to be divided among Treasury notes, bonds, and stocks. The rules for administration of the fund require that at least \$3 million be invested in each type of investment, at least half the money be invested in Treasury notes and bonds, and the amount invested in bonds not exceed twice the amount invested in Treasury notes. The annual yields for the various investments are 7% for Treasury notes, 8% for bonds, and 9% for stocks. How should the money be allocated among the various investments to produce the largest return?

*Hint.* This looks like a three-variable linear program, where  $x$  is the amount invested in Treasury notes,  $y$  the amount in bonds, and  $z$  the amount in stocks. Assume that all money is invested, so  $z = 30 - x - y$ . Thus you need only do a two-variable LP problem.

You could use the simplex method, but I advise against it.

8

8

8. (10 Points) Three salespeople have to travel to three clients on a certain day. Each salesperson will go to exactly one client. The miles from each salesperson to each client are given below:

	Client A	Client B	Client C
Salesperson 1	72	99	88
Salesperson 2	23	30	35
Salesperson 3	51	59	84

Who should go where to minimize total mileage traveled by the sales team?

9

9

9. (10 Points) In a certain town it is always either sunny or rainy. On any given day the probability of rain is  $\frac{1}{3}$  if the previous day was sunny and  $\frac{1}{2}$  if the previous day was rainy. What is the eventual distribution of sunny and rainy days?

**10**

**10**

**10.** (10 Points) A student derives utility (happiness) from consuming burritos and chips according to the function

$$U(x, y) = 10x^{1/2}y^{2/5},$$

where  $x$  is the number of burritos consumed and  $y$  is the number of baskets of chips consumed every week. Burritos cost \$2 apiece and baskets of chips cost \$3 apiece. If the student has a fixed budget of \$60 with which to buy his Mexican food, what quantities of burritos and chips should he buy to maximize utility?

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