Name: _____ ID#: _____

Midterm II

Math 20
Introduction to Linear Algebra
and Multivariable Calculus

22 November 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.

—Handbook for Students

Problem Number	Possible Points	Points Earned
1	12	
2	8	
3	20	
4	13	
5	12	
6	10	
Total	75	

1. (12 Points) Suppose that

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} = 7.$$

Find the following, with justification.

(i)

$$\det \begin{bmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & k \end{bmatrix}$$

(ii)

$$\det \left(3 \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \right)$$

(iii)

$$\det \begin{bmatrix} d & e & f \\ g & h & k \\ a & b & c \end{bmatrix}$$

(iv)

$$\det \begin{bmatrix} a & b & c \\ d+g & e+h & f+k \\ g & h & k \end{bmatrix}$$

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2. (8 Points) Suppose B is a square matrix with $B^2 = B$. What can you say about $\det(B)$?

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3. (20 Points) Let

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. (The next page is blank if you want extra scratch paper).

Hint. The determinant of this matrix is zero, so this means that 0 is one of the eigenvalues of A. So the characteristic polynomial won't have a constant term.

Reminder. In this and all problems in which you use Gaussian Elimination (row reduction), label each operation to receive partial credit.

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4. (13 Points) Suppose that the probability that the child of a college-educated parent also becomes college-educated is 0.75, and that the probability that the child of a non-college-educated parent becomes college-educated is 0.35.

(a) (5 points) Assuming that the information about describes a Markov chain, write a transition matrix for this situation.

(b) (3 points) If 30% of parents are college-educated, what proportion of the population will be college-educated next generation?

(c) (5 points) Without any knowledge of the present proportion of college-educated parents, determine the eventual proportion of college-educated people.

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5. (12 Points) Consider the set of vectors in $\mathbb{R}^4$

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \qquad \mathbf{x}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} \qquad \mathbf{x}_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix}$$

Find an orthonormal set of vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ with the same span. (Use the Gram-Schmidt orthogonalization process.)

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6. (10 Points) Consider the three points $(1, 1)$, $(2, 1)$, and $(3, 3)$ in the plane. Clearly these points are not on a single line. That means there do not exist β_0 and β_1 such that

$$\beta_0 + 1 \cdot \beta_1 = 1$$
$$\beta_0 + 2 \cdot \beta_1 = 1$$
$$\beta_0 + 3 \cdot \beta_1 = 3$$

(The notation is a little odd to think about lines, but at least makes a vector $\boldsymbol{\beta}$. Notice that β_0 is the "y-intercept" of the supposed line and β_1 its slope).

    Find $\hat{\beta}_0$ and $\hat{\beta}_1$ which form the least-squares solution to this system of linear equations. You will have found the line which fits this data the closest.

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