

Name: _____ ID#: _____

Final Examination

Math 20
Introduction to Multivariable Calculus and Linear Algebra

January 23, 2006

Rules:

- This is a three-hour exam.
- Calculators are not allowed.
- Unless otherwise stated, show all of your work. Full credit may not be given for an answer alone.
- You may use the backs of the pages or the extra pages for scratch work. *Do not unstaple or remove pages as they can be lost in the grading process.*
- Please do not put your name on any page besides the first page. If you like, you may put your ID number on the top of each page you write on.

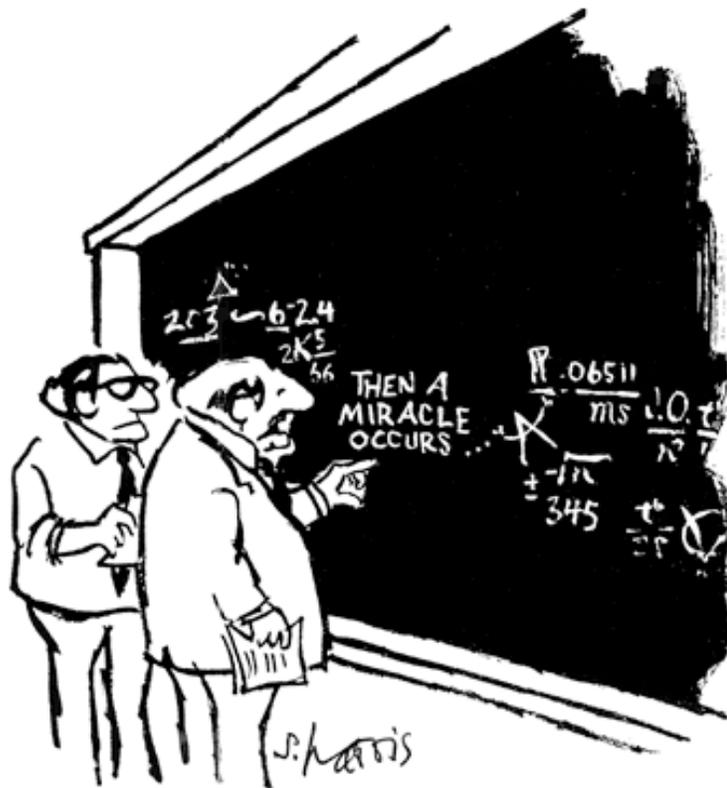
Hints:

- Read the entire exam to scan for obvious typos or questions you might have.
- Budget your time so that you don't run out.
- Problems may stretch across several pages.
- Relax and do well!

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.

—Handbook for Students

Problem Number	Possible Points	Points Earned
1	15	
2	15	
3	16	
4	8	
5	10	
6	9	
7	10	
8	9	
9	8	
Total	100	



"I think you should be more explicit here in step two."

Remember to show your work!

Part I. Techniques

1. (15 Points) For $n \times n$ matrices A and B , define

$$[A, B] = AB - BA.$$

Notice $AB = BA$ if and only if $[A, B] = 0$.

For

$$X = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Find

(i) $[H, X]$

(ii) $[H, Y]$

(iii) $[X, Y]$

2

2

2. (15 Points) Let

$$f \begin{pmatrix} x \\ y \end{pmatrix} = 4xy - 2x^4 - y^2$$

(a) Find $\frac{\partial f}{\partial x} \begin{pmatrix} x \\ y \end{pmatrix}$ and $\frac{\partial f}{\partial y} \begin{pmatrix} x \\ y \end{pmatrix}$.

(b) Find the critical points of f .

(continued)

2

2

(c) For each critical point, decide if it's a local maximum or a local minimum.

3

3

3. (16 Points) Let

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

(a) Is 3 an eigenvalue for A ?

(b) Find an orthogonal matrix Q and a diagonal matrix Λ such that

$$A = Q\Lambda Q^T$$

4**4**

4. (8 Points) Let $W \subset \mathbb{R}^5$ be the subset

$$W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \text{Span}\left\{\begin{bmatrix} 1 \\ 1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \\ 8 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 3 \\ 2 \end{bmatrix}\right\}.$$

Find a basis for W and determine its dimension.

Part II. Applications

5

5

5. (10 Points) Consider a simple economy consisting of two sectors: goods and services. To produce its output, the goods sector consumes $\frac{3}{4}$ of the goods and $\frac{1}{5}$ of the services. The services sector consumes $\frac{1}{4}$ of the goods and $\frac{4}{5}$ of the services.

(a) (4 points) Find the exchange matrix A of this economy.

(b) (6 points) Find an equilibrium production vector \mathbf{p} that satisfies $A\mathbf{p} = \mathbf{p}$ and $\mathbf{p} \geq \mathbf{0}$ with at least one component positive.

6. (9 Points)

I'm getting ready for my annual Super Bowl party and need to order my provisions. Last year my budget was \$120, and I spent \$40 on nachos and \$80 on chicken wings. Economics tells me that I maximized my personal utility function of nachos and wings, which we'll assume is of the form



$$u(x, y) = x^\alpha y^{1-\alpha}.$$

α is some number between 0 and 1. I'm just a consumer, though; I don't know my personal value of α . Can you tell me?

7. (10 Points) In designing a new airline route, a company is considering two types of planes, types A and B . Each type A plane can carry 40 passengers and requires two mechanics for servicing; each type B plane can carry 60 passengers and requires three mechanics for servicing. Suppose that the company must transport at least 300 people daily and that insurance rules for the size of the hanger allow no more than 18 mechanics on the payroll. If each type A plane costs \$10,000,000 and each type B plane costs \$12,000,000, how many planes of each type should be bought to minimize the cost?

- (i) (3 points) State the problem in linear programming formalism. What are the decision variables? What is the objective? What are the constraints?
- (ii) (3 points) Draw the feasible set.

(continued)

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7

(iii) (4 points) Solve the problem.

8. (9 Points) Player R has a nickel and dime with him. He chooses one of the coins and player C must guess R 's choice. If C guesses correctly, he keeps the coin; if he guesses incorrectly, he must give R an amount equal to the coin shown.

(a) (3 points) Write down the payoff matrix for this game.

(b) (4 points) In repeated rounds of the game how often should each player adopt each strategy?

(c) (2 points) Which player does the game favor?

9. (8 Points) The subscription department of a magazine sends out letters to a large mailing list inviting subscription for the magazine. Some of the people receiving this letter already subscribe to the magazine, while others do not. From this mailing list, 60% of those who already subscribe will subscribe again, while 25% of those who do not now subscribe will subscribe.

(a) (3 points) Find the transition matrix A of this Markov process.

(b) (5 points) After many letters sent out to the same mailing list, what portion of the members will be subscribed?

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