Name: ________________________________ ID#: ____________________

Midterm I

Math 20
Introduction to Linear Algebra and Multivariable Calculus

October 17, 2005

Rules:

- This is a one-hour exam.
- Calculators are not allowed.
- Unless otherwise stated, show all of your work. Full credit may not be given for an answer alone.
- You may use the backs of the pages or the extra pages for scratch work. *Do not unstaple or remove pages as they can be lost in the grading process.*
- Please do not put your name on any page besides the first page. If you like, you may put your ID number on the top of each page you write on.

Hints:

- Read the entire exam to scan for obvious typos or questions you might have.
- Budget your time so that you don't run out.
- Problems may stretch across several pages.
- Relax and do well!

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.
—*Handbook for Students*

1. (6 Points) Let

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Find

(i) $\mathbf{v} - 2\mathbf{w}$

Answer: ________

(ii) $\mathbf{v} \cdot \mathbf{w}$

Answer: ________

(iii) $\|\mathbf{v}\|$

Answer: ________

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2. (6 Points) Let

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Find AB and BA.

$$AB = \begin{bmatrix} & \\ & \end{bmatrix} \qquad BA = \begin{bmatrix} & \\ & \end{bmatrix}$$

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3. (13 Points)

(i) Let $u\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x^2y + xy^3 + 3$. Find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, and $\frac{\partial u}{\partial z}$.

$\frac{\partial u}{\partial x} =$ ______________________________

$\frac{\partial u}{\partial y} =$ ______________________________

$\frac{\partial u}{\partial z} =$ ______________________________

(continued)

(ii) Let $u\begin{pmatrix} x \\ y \end{pmatrix} = x^y$. Find $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$.

$$\frac{\partial u}{\partial x} = \underline{\hspace{8cm}}$$
$$\frac{\partial u}{\partial y} = \underline{\hspace{8cm}}$$

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4. (8 Points)

(a) Find the matrix corresponding to the linear transformation which reflects the plane in the x_1 axis. Call this matrix A.

$$A = \begin{bmatrix} \quad & \quad \end{bmatrix}$$

(b) Find the matrix corresponding to the linear transformation which reflects the plane in the x_2 axis. Call this matrix B.

$$B = \begin{bmatrix} \quad & \quad \end{bmatrix}$$

(continued)

(c) Find the matrix corresponding to the linear transformation which rotates the plane by 180° about the origin. Call this matrix C.

$$C = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

(d) What is the product AB?

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5. (6 Points) Let V be the following subset of $\mathbb{R}^3$:

$$V = \left\{ \mathbf{x} \in \mathbb{R}^3 \;\middle|\; \mathbf{x} = \begin{bmatrix} s+t \\ 2s+t \\ s+2t \end{bmatrix} \text{ where } s,\, t \in \mathbb{R} \right\}$$

We will show that V is a subspace of $\mathbb{R}^3$ in two ways:

(i) Show that V is the span of two vectors: In other words, there exist vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ such that

$$V = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}.$$

$\mathbf{v}_1 =$ ________ $\mathbf{v}_2 =$ ________

(ii) Show that V is the "perp space" to a vector $\mathbf{w}$: In other words, there exists a vector $\mathbf{w} \in \mathbb{R}^3$ such that

$$V = \mathbf{w}^\perp = \left\{ \mathbf{x} \in \mathbb{R}^3 \;\middle|\; \mathbf{x} \cdot \mathbf{w} = \mathbf{0} \right\}.$$

$\mathbf{w} =$ ________

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6. (11 Points)

(i) Show

$$\lim_{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \longrightarrow \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]} \frac{x}{x^2 + y^2}$$

does not exist by finding a line through the origin along which the limit does not exist.

(ii) Show

$$\lim_{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \longrightarrow \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]} \frac{xy}{x^2 + y^2}$$

does not exist by finding two lines through the origin along which the limits exist but disagree.

(continued)

(iii) Now we will show

$$\lim_{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \longrightarrow \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]} \frac{xy^2}{x^2 + y^2} = 0$$

(a) Show that

$$|x| \leq \sqrt{x^2 + y^2} \qquad\qquad |y| \leq \sqrt{x^2 + y^2}$$

for all real numbers x and y. (This is easier than it looks; you don't need any fancy theorems. Why not square both sides?)

(b) Use (a) to show that

$$\left| \frac{xy^2}{x^2 + y^2} \right| \leq \sqrt{x^2 + y^2}.$$

(You can do this part even if you didn't get part (a).)

(c) What is the limit of the right-hand side of this inequality as x and y both tend to zero? (Just say the answer, no need to prove it.)

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