Name: _____ ID#: _____

Midterm III

Math 20
Introduction to Linear Algebra and Multivariable Calculus

December 16, 2005

Rules:

- This is a one-hour exam.
- Calculators are not allowed.
- Unless otherwise stated, show all of your work. Full credit may not be given for an answer alone.
- You may use the backs of the pages or the extra pages for scratch work. *Do not unstaple or remove pages as they can be lost in the grading process.*
- Please do not put your name on any page besides the first page. If you like, you may put your ID number on the top of each page you write on.

Hints:

- Read the entire exam to scan for obvious typos or questions you might have.
- Budget your time so that you don't run out.
- Problems may stretch across several pages.
- Relax and do well!

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.
—Handbook for Students

Problem Number	Possible Points	Points Earned
1	5	
2	5	
3	10	
4	15	
5	15	
Total	50	

1 1

1. (5 Points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation whose standard matrix is

$$A = [T]_{\mathcal{E}} = \begin{bmatrix} 2 & 8 \\ 4 & -2 \end{bmatrix}$$

Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

Find $[T]_{\mathcal{B}}$.

_____ / 5

2. (5 Points) It is March 1943, and Rear Admiral Kimura Masatomi is trying to move a convoy from Rabaul, on the northeastern tip of the island of New Britain, to Lae, just west of New Britain on the island of New Guinea.

He can take the northern route or the southern route around New Britain. Both routes would take three days to reach Lae but visibility will be poor along the northern route due to bad weather.

The Allies have learned of this plan (but not Masatomi's choice). Their strategy is to find the convoy by air reconnaissance and bomb it until it reaches Lae. They need to decide whether to search along the northern route or along the southern route. Clearly they will have more time to bomb if they choose correctly, and remember the visibility along the northern route is poor. General George Kenney's staff estimate that depending on the routes taken, the possible number of days of bombing can be given by this table:

		Japanese	
		North	South
Allies	North	2	2
	South	1	3

What should Kenney and Masatomi decide and what will be the result?

_____ / 5

3 3

3. (10 Points) During a penalty kick in soccer, the shooter S kicks the ball towards the goal while the goalie G defends the goal.

S may aim to the left or to the right of the goal, G may dive to his left or right. (So if S aims left, and G dives right, ball and goalkeeper should be close together.) Suppose G has a 50% chance of stopping a shot on target if he dives the way that S kicks. Further suppose that the kicker is more accurate when shooting to the right—he is then on target 90% of the time, but only 70% of the time when shooting left.

(a) Find the payoff matrix for this game.

(b) With what probability should the kicker decide to shoot to the left?

_____ / 10

4 4

4. (15 Points) In the back room of a book store there are a large number of copies of books. There is a novel by Nabokov and one by Updike. There are also calculus books: one by Stewart, one by Thomas, and one by Gottlieb. The novels are each one inch thick, and the calculus books are each two inches thick.

Let r_n be the number of ways of arranging books in a stack n inches high.

(i) Find r_0, r_1, and r_2.

(ii) Show that $\{r_n\}$ satisfies the difference equation

$$r_n = 2r_{n-1} + 3r_{n-2}$$

for $n \geq 2$.

4 4

(iii) Solve for r_n in terms of n.

(iv) How many ways are there to stack the books three feet high?

_____ / 15

5 5

5. (15 Points) A department is trying to schedule teaching fellows for a course. The teaching fellows are polled and have given their preferences of section time. TF A prefers teaching at 9AM, 10AM, 11AM, 12PM in that order. TF B prefers teaching at 10, 12, 11, and 9. TF C prefers 10, 9, 12, and 11.

Find an assignment of teaching fellows to section times that maximizes satisfaction. (Note: you will have to invent a dummy TF who will take the leftover time slot.)

_____ / 15

(This page intentionally left blank. You can use it for scratch work.)