

The Dirichlet and Fourier Kernels

Math212a

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The Dirichlet kernel D_n is given by

$$D_n := 1 + \sum_{k=1}^n \cos kt = \frac{\sin(n + \frac{1}{2})t}{\sin \frac{t}{2}}$$

where the value at $t = 0$ is $2n + 1$. Clearly

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} D_n(t) dt = 1$$

for all n . As n increases, D_n becomes more and more oscillatory outside any fixed interval about the origin, but its amplitude does not tend to zero there, in contrast to the Fejer kernels. The Fourier kernel F_n is defined as

$$F_n(t) := \frac{\sin(n + \frac{1}{2})t}{\frac{t}{2}}.$$

In the interval $[-2, 2]$ the Fourier kernel is very close to the Dirichlet kernel:



