

# Questionable sums

Math 212

Sept.19, 2000

Let

$$a := 1 - 1 + 1 - \dots.$$

Then

$$a = 1 - (1 - 1 + 1 - 1 + \dots) = 1 - a$$

so

$$a = \frac{1}{2}.$$

Let

$$s = 1 - 2 + 3 - 4 + 5 - 6 + \dots.$$

Then

$$\begin{aligned} s &= 1 + (-2 + 3 - 4 + \dots) \\ &= 1 - (2 - 3 + 4 - 5 + \dots) \\ &= 1 - (1 - 1 + 1 - 1 + \dots) - (1 - 2 + 3 - 4 + \dots) \\ &= 1 - a - s \end{aligned}$$

so

$$s = 1 - \frac{1}{2} - s$$

or

$$s = \frac{1}{4}.$$

Let

$$Z := 1 + 2 + 3 + 4 + 5 + \dots.$$

Then

$$Z - s = 0 + 4 + 0 + 8 + 0 + 12 + \dots = 4Z$$

so

$$3Z = -s$$

or

$$Z = -\frac{1}{12}.$$

Euler: "ich glaube, dass jede series einem bestimmten Wert haben müsse. Um aber allen Schwierigkeiten worden zu begegnen, so sollte dieser werten nicht mit dem Namen Summe belegen...."

Hardy *Divergent series*:(p.5) " it does not occur to a modern mathematician that a collection of symbols should have a 'meaning' until one has been assigned to it by definition. [This] was not a triviality even to the greatest mathematicians of the eighteenth century. They had not the habit of definition: it was not natural to them to say, in so many words, 'by X we *mean* Y. ... mathematicians before Cauchy asked not 'How shall we *define*  $1 - 1 + 1 - \dots$ ' but ' What *is*  $1 - 1 + 1 - \dots$  ?'."