

Summary of Lecture 7, Lebesgue measure.

Math 212a

October 16, 2001

We covered the material in the handout *measuretheory*. This included:

- the definition of Lebesgue outer and inner measures for subsets of \mathbf{R} ,
- the definition of a Lebesgue measurable set as one whose inner measures coincide,
- A set A is Lebesgue measurable if and only if for any $\epsilon > 0$ there is an open set $U \supset A$ and a closed set $F \subset A$ with $m^*(U \setminus F) < \epsilon$.
- A set E is measurable in the sense of Caratheory if and only if for any set A we have

$$m^*(A) = m^*(A \cap E) + m^*(E \setminus E).$$

- measurability in the sense of Caratheodory and measurability in the sense of Lebesgue are the same for subsets of the real line.
- Countable additivity follows from the Caratheodory definition.