1. Prove that $L^\infty([0,1])$ is not separable.

2. Give an example that $f^j \rightharpoonup 0$ weakly in $L^2(\mathbb{R}^n)$, $f^j \to 0$ strongly in $L^{3/2}(\mathbb{R}^n)$ but not strongly in $L^2(\mathbb{R}^n)$. Prove your statement.

3. Suppose $f^j \to f$ a.e. for $x \in [0,1]$. Furthermore, we assume that $\|f^i\|_{L^2[0,1]} \leq 1, \|f\|_{L^2[0,1]} \leq 1$. Prove that

$$\lim_{j\to\infty} \|f^j - f\|_p = 0, \qquad 1 \leq p < 2$$

Does the conclusion hold for $p = 2$? Does it follow that $f^j \rightharpoonup f$ weakly in $L^2[0,1]$? Either prove them or give counter examples.

4 If $f^j \to f$ strongly in $L^2([0,1])$. Does it follow that $f^j(x) \to f(x)$ a.e.? Is there a subsequence so that $f^{k_j}(x) \to f(x)$ a.e.?

5. Suppose $f^j(x) \to f(x)$ a.e. for $x \in [0,1]$. Furthermore, we assume that $\|f^j\|_2 \leq 1$. Is it true that

$$\lim_{j\to\infty} \|f^j - f\|_1 = 0 \ ?$$

Prove it or give a counter example.

Please type it or write it very clearly (and readable).