MATH 213B: MIDTERM EXAM
DUE IN CLASS ON MONDAY, MARCH. 10TH

YOU MAY CONSULT YOUR NOTES AND ANY BOOKS, BUT YOU MAY NOT TALK TO ANYONE BUT ME ABOUT THIS EXAM.

1) Let $\mathfrak{H} = \{(z_1, z_2) \in \mathbb{C}^2 \mid |z_1| < |z_2| < 1\}$. $\mathfrak{H}$ is called the Hartogs triangle.

(a) (10 pts) Prove that $\mathfrak{H}$ is a domain of holomorphy.
(b) (10 pts) Let f be any function holomorphic on a neighborhood of $\mathfrak{H}$. Prove that f has a holomorphic extension to the bidisk $D_1^2(0)$.
(c) (5 pts) Explain why the statements in (a) and (b) do not contradict each other.

2) (10 pts) Give an example of a domain $\Omega \subset \mathbb{C}^n$ with at least one point of strong Levi pseudoconcavity (i.e. a point where the Levi form is negative definite). Justify your answer.

3) (15 pts) Let $\Omega \subset \mathbb{C}^n$ be a domain. A point $P \in \partial\Omega$ is called essential if there exists some function h holomorphic on Ω such that h cannot be extended past P. Give an example of a domain $\Omega \subset \mathbb{C}^n$, where $n \geq 2$, which has (at least) one essential point P that is **not a point of weak convexity** and of a holomorphic function h which cannot be extended past P. Justify your answer.