Math 21a F99 answers for Hourly 2 practice problems

1. a) $L(x, y, z) = 2x + 2y + 2z - 3$. b) $L(x, y, z) = y + z$.

2. 12 (Note: Since $y_r = 0$ at $r = 1$ and $s = -1$, you don't need to compute any y derivatives when using the Chain Rule.)

3. $\pi + 1$. ($w' = 4t \tan^{-1}(t) + 1$ at $t = 1$).

4. $\nabla f = (3, 2, -4)$.

5. 2

6. $\mathbf{u} = \frac{1}{\sqrt{3}}(1, 1, 1)$.

7. 0.

8. $2x + 2y + z - 4 = 0$.

9. $x - y + 2z - 1 = 0$.

10. $2\sqrt{2}$.

11. Absolute max: 1 at (0, 0); absolute min: -5 at (1, 2). Note that this is a tricky question as you have to check the corner points by hand.

12. Absolute max: 4 at (2, 0); absolute min: $3/\sqrt{2}$ at $(3, \pi/4)$, $(3, -\pi/4)$, $(1, \pi/4)$ and $(1, -\pi/4)$.

13. Local min at (1, -2); saddle point at (-1, -2).

14. Length in the x-direction: $4\sqrt{2}$; length in the y-direction: $3\sqrt{2}$.

15. (3/2, 2, 5/2).

16. (0, 0, 2) and (0, 0, -2).

17. (±4/3, -4/3, -4/3).

18. $\begin{pmatrix} -30x + \dfrac{2-2x^2}{(1+x^2)^2} & 1 \\ 1 & 0 \end{pmatrix}$

19. $L(x, y, z) = x + y - z - 1$.

20. $w = (b^2\ V/(ac))^{1/3}$; $d = (c^2\ V/(ab))^{1/3}$; $h = (a^2\ V/(bc))^{1/3}$.

21. A normal vector to the surface is the gradient of $xz^2 - yz + \cos(xy) - 1$ which, at (0, 0, 1), is the vector $\mathbf{v}$ = (1, -1, 0). Meanwhile, $\mathbf{r}(1)$ = (0, 0, 1) and $\mathbf{r}'(1)$ = (1, 1, 1) which has zero dot product with the normal vector $\mathbf{v}$. Thus, $\mathbf{r}'(1)$ lies in the tangent plane to the surface.

22. Set $z(x, y) = x^3 + y^3 - 9xy + 27$ as a function on $\mathbb{R}^2$. Then, maxima and minima can only occur at points where ∇z is zero. Since $\nabla z = (3x^2 - 9y, 3y^2 - 9x, 0)$, this occurs where $x^2 = 3y$ and $y^2 = 3x$. Thus, at (x, y) = (0, 0) and (x, y) = (3, 3). The second derivative test for z'' at the point (0, 0) finds z'' to have only the off diagonal entries non-zero. (These equal 9). Thus, $\det(z''(0, 0)) < 0$ so (0, 0) is a saddle. At (3, 3), $\det(z'') = 243 > 0$ and $\mathrm{trace}(z'') = 36$ so (3, 3) is a local minimum.

23. 1.

24. (3 ln2)/2.

25. 2.

26. (e - 2)/2.

27. 12.

28. -48

29. 16.

30. 1/75