

## Math 21a F00 Hourly 2 practice problems

What follows are thirty mostly straightforward problems about the material that will be on the second hourly; answers are provided elsewhere at this website. In this regard, note that the second hourly will cover material in Chapter 2, Section 4.4, Sections 3.1-3.2 and the Lagrange Multiplier supplement. The second hourly will comprise roughly six or seven problems with most about material directly touched on in the class or the reading. However, one or more may probe the limits of your understanding by asking about issues have not been explicitly mentioned. (The problems below are not necessarily representative of the latter sort.) Note that there is also a 'practice' second hourly exam elsewhere at this website which is meant to give you an indication of what the second hourly will actually look like.

1. Find the linear approximation to the function  $f(x, y, z) = xy + yz + zx$  at
  - a) The point  $(1, 1, 1)$ .
  - b) The point  $(1, 0, 0)$ .
2. Find  $\frac{\partial w}{\partial r}$  at  $(r, s) = (1, -1)$  if  $w = (x + y + z)^2 \cos(\pi(x + 4y)/3)$  and  $(x, y, z)$  are the following functions of  $r$  and  $s$ :  $x = r - s$ ,  $y = \cos(r + s)$  and  $z = \sin(r + s)$ .
3. Find  $\frac{dw}{dt}$  at  $t = 1$  for  $w = 2y e^x - \ln z$  where  $x = \ln(t^2 + 1)$ ,  $y = \tan^{-1} t$  and  $z = e^t$ .
4. Find  $\nabla f$  at  $(1, 1, 1)$  for  $f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x$ .
5. Find the directional derivative at  $(0, 0, 0)$  in the direction of  $\mathbf{u} = \frac{1}{3}(2, 1, -2)$  for the function  $f(x, y, z) = 3 e^x \cos(yz)$ .
6. Find the unit vector giving the direction of most rapid increase at the point  $(1, 1, 1)$  for the function  $f(x, y, z) = \ln(xy) + \ln(yz) + \ln(xz)$ .
7. Estimate the amount by which the function  $g(x, y, z) = x + x \cos(z) - y \sin(z) + y$  will change if one moves from  $(2, -1, 0)$  a distance 0.02 units towards the point  $(0, 1, 2)$ .
8. Find the equation for the tangent plane at  $(0, 1, 2)$  to the surface where  $(x, y, z)$  obeys the equation  $\cos(\pi x) - x^2 y + e^{xz} + yz - 4 = 0$ .
9. Find the equation for the tangent plane at  $(1, 2, 1)$  to the surface where  $z^2 = y - x$ .

10. The derivative of a certain function  $f(x, y, z)$  at a certain point  $P$  is greatest in the direction of the vector  $(1, 1, -1)$  and in this direction, the derivative is  $2\sqrt{3}$ . What is the derivative at  $P$  of this same function in the direction of the vector  $(1, 1, 0)$ ?
11. Find the maximum and minimum values of  $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$  on the triangle in the first quadrant whose boundaries lie where  $x = 0$ ,  $y = 2$  and  $y = 2x$ .
12. Find the maximum and minimum values of  $f(x, y) = (4x - x^2) \cos(y)$  where  $(x, y)$  obey the conditions  $1 \leq x \leq 3$  and  $-\pi/4 \leq y \leq \pi/4$ .
13. Find the maxima, minima and saddle point of a function  $f(x, y)$  (if any), given that the  $f$ 's partial derivatives are  $f_x = 9x^2 - 9$  and  $f_y = 2y + 4$ .
14. Find the dimensions of the rectangle of greatest area that has sides parallel to the  $x$ - $y$  coordinate axes and fits in the ellipse where  $x^2/16 + y^2/9 = 1$ .
15. Find the point on the plane  $x + 2y + 3z = 13$  which is closest to the point  $(1, 1, 1)$ .
16. Find the points on the surface  $z^2 = xy + 4$  closest to the origin.
17. The surface of a space probe is the shape of the ellipsoid  $4x^2 + y^2 + 4z^2 = 16$ . Meanwhile, the temperature on the surface of this probe is given by  $T(x, y, z) = 8x^2 + 4yz - 16z + 600$ . Find the hottest point on the surface.
18. Find the  $2 \times 2$  matrix  $f''$  of 2<sup>nd</sup> order partial derivatives of  $f(x, y) = x + xy - 5x^3 + \ln(x^2 + 1)$ .
19. Find the best linear approximation to  $f(x, y, z) = xy + 2yz - 3xz$  at  $(1, 1, 0)$ .
20. A closed, rectangular box, when viewed from the front, has height  $h$ , width  $w$  and depth  $d$ . The cost of the material used in the box is  $a$  cents/cm<sup>3</sup> for the top and bottom,  $b$  cents/cm<sup>3</sup> for the front and back, and  $c$  cents/cm<sup>3</sup> for the remaining sides. In terms of  $V$ ,  $a$ ,  $b$ , and  $c$ , find the dimensions  $h$ ,  $w$  and  $d$  which minimize the cost of the box.
21. Show that the curve  $\mathbf{r}(t) = (\ln(t), t \ln(t), t)$  is tangent to the surface  $xz^2 - yz + \cos(xy) = 1$  at the point  $(0, 0, 1)$ .
22. Show that the only possible maxima and minima of  $z$  on the surface  $z = x^3 + y^3 - 9xy + 27$  occur at  $(0, 0, 27)$  and  $(3, 3, 0)$ . Show that  $(0, 0, 27)$  is neither a maximum nor a minimum. Is  $(3, 3)$  a local maximum or minimum?

23. Integrate  $f(x, y) = x + y + 1$  over the region where  $-1 \leq x \leq 1$  and  $-1 \leq y \leq 0$ .
24. Integrate  $f(x, y) = x/y$  over the region in the first quadrant cut out by the four lines whose equations are  $y = x$ ,  $y = 2x$ ,  $x = 1$  and  $x = 2$ .
25. Integrate  $f(x, y) = \sin(y)/y$  over the region where  $0 \leq x \leq \pi$  and  $x \leq y \leq \pi$ . (Hint: Be careful when you chose the order in which to do this integral.)
26. Integrate  $f(x, y) = x^2 e^{xy}$  over the region where  $0 \leq y \leq 1$  and  $y \leq x \leq 1$ .
27. Find the area of the region where  $0 \leq y \leq 6$  and  $y^2/3 \leq x \leq 2y$ .
28. Find the integral of  $4 - 5y^2$  over the region where  $0 \leq x \leq 4y$ ,  $0 \leq y \leq 2$ .
29. Find the volume of the region where  $0 \leq x \leq 3$ ,  $0 \leq y \leq 2$  and  $0 \leq z \leq 4 - y^2$ .
30. Integrate the function  $z^2xy$  over the region where  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$  and  $0 \leq z \leq xy$ .