

## Math 21a Practice Hourly 2

(Fall 2000)

What follows is a model of the second hourly exam (it is more or less the exam given last fall in this course). The upcoming exam on Wednesday, November 15 will be roughly similar to this one in length and difficulty. To study for the upcoming exam, first work the suggested problems for the Chapters 2, 3.1-3.2 and 4.4 and in the Supplement on Lagrange Multipliers. The exam will cover solely material from these readings. The list of suggested problems is posted on the course website. You should also work the practice problems for Hourly 2 supplied elsewhere at this website. After you have worked the suggested problems and checked your answers with those in the book, and worked the practice problems and checked your answers, try taking this exam as practice for the real thing. In this regard, note that you will have two hours for the real thing, but we hope that most people finish under the two hour limit. The answers to this practice hourly are provided elsewhere at the website.

By the way, in the real exam, each problem will be printed on a separate page and you will be asked to provide your answers on that page and to use the back of that page if you need more space to show your work.

Remember that the exam on *Wednesday, November 15* is from 7-9pm in *Lecture Halls C and D of the Science Center*. Please come a few minutes early as the exam will start promptly at 7pm.

1) \_\_\_ 2) \_\_\_ 3) \_\_\_ 4) \_\_\_ 5) \_\_\_ 6) \_\_\_ 7) \_\_\_ : Total \_\_\_\_\_

Name: \_\_\_\_\_

Circle the name of your Section TA:

Allcock • Chen • Karigiannis • Knill • Liu • Rasmussen • Rogers • Taubes • Winter • Winters

**Instructions:**

- Print your name in the line above and circle the name of your section TF.
- Answer each of the questions below in the space provided. If more space is needed, use the back of the facing page.
- Please write neatly. Answers which are deemed illegible by the grader will not receive credit.
- No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students. Use only your brain and a pencil.
- You have 2 hours to complete your work..

*In agreeing to take this exam, you are implicitly accepting Harvard University's honor code.*

1. Consider the surface where the function  $g(x, y, z) = y^2 - 3xy + zx + 2x - z$  is zero.
  - a) Find the tangent plane to this surface at  $(-1, 0, -1)$ .
  - b) Find a point P on this surface where the tangent plane is given by the equation  $x = y$ .  
Explain your reasoning.
  - c) Find the best linear approximation to this function  $g(x, y, z)$  at the point  $(-1, 0, -1)$ .
  - d) Find a direction (a unit vector) in which the directional derivative of  $g$  at  $(-1, 0, -1)$  is zero.
2. Suppose that the moon is modeled by the ball where  $x^2 + y^2 + z^2 \leq 1$ . If the temperature of a point  $(x, y, z)$  in or on the moon at a particular time is given by the function

$$T(x, y, z) = 50(1 - x^2 - y^2 - z^2) + 10(\sqrt{3}x + z),$$

then find:

- a) The  $(x, y, z)$  coordinates of the hottest of the points on the surface of the moon.
- b) The  $(x, y, z)$  coordinates of the points in or on the moon where temperature is greatest.
- c) The  $(x, y, z)$  coordinates of the points in or on the moon where the temperature is least.

In all cases, make sure you justify your conclusions.

3. The positions of a rat and a snail on the x-y plane are given, respectively by

$$\mathbf{r}(t) = (1 + t) \mathbf{i} + (t^2 + t) \mathbf{j} \quad \text{and} \quad \mathbf{s}(t) = \cos(t) \mathbf{i} + (t^3 - t) \mathbf{j},$$

where  $t$  is time measured in seconds.

- Give the velocity and acceleration vectors for the rat at  $t = 0$ .
  - Give the velocity and acceleration vectors for the snail at  $t = 0$ .
  - The temperature of any point  $(x, y)$  on the plane is position dependent and thus given by a function,  $T(x, y)$ , measured in degrees. In particular, note that the temperature at any given point doesn't change with time. However, as the rat and snail are moving, they feel the temperature change. For example, at  $t = 0$  the rat feels the temperature increase at the rate of 3 degrees per second, while the snail feels the temperature decrease at the rate of 1 degree per second. With the preceding understood, compute  $\nabla T$  at the point  $(1, 0)$ .
4. Let  $f(x, y) = x^2 y - 4xy + y^3/3$ .
- Find all stationary points of  $f$ .
  - Identify the points found in Part a as local maxima, minima or saddle points. Justify your answers here.
  - Find the directions of maximum increase and decrease of  $f$  at  $(1, -3)$ .
  - Find the tangent vector to the level curve of  $f$  through  $(2, 4)$ .
5. Use the best linear approximation to  $f(x, y) = e^x y^{1/2}$  at a convenient point to estimate the value of  $f$  at  $(0.1, 25.3)$ .
6. Integrate the function  $f(x, y) = x^3 y$  over the region where  $x \geq 0, y \geq 0$  and  $x^2 + y^2 \leq 1$ .
7. Calculate  $\iint_R 2e^{x^2} dA$  where  $R$  is the triangle where  $0 \leq y \leq 1$  and  $y \leq x \leq 1$ . Thus, the vertices of  $R$  are  $(0, 0)$ ,  $(1, 0)$  and  $(1, 1)$  in the x-y plane.