

Answers to Math 21a Hourly 1

(Fall 2000)

1. a) The line traversed can be parametrized by $t \rightarrow (-2 + 3t, 8 - 2t, -6 + 2t)$.
 b) The surface is reached when the z-coordinate, $-6 + 2t$ is zero. Thus, $t = 3$ and the coordinates of the point in question are $(7, 2, 0)$.
 c) The closest point has distance 6 from the origin. (The point is $(4, 4, -2)$.)

2. a) The bug's velocity is $\mathbf{r}' = 2 \left(\frac{1}{t}, t, -\sqrt{2} \right)$.
 b) The bug's speed is $|\mathbf{r}'| = 2 \left(\frac{1}{t} + t \right) = \sqrt{4 \left(\frac{1}{t^2} + t^2 + 2 \right)}$.
 c) The path length is the integral from 1 to 2 of $|\mathbf{r}'|$ which is $3 + 2 \ln(2)$.
 d) The component is $\frac{1}{3}(1 - 2\sqrt{2})(1, 1, 1)$.

3. a) False, as $\mathbf{w} = 0$ when either \mathbf{u} or \mathbf{v} is zero, or when $\mathbf{v} = r \mathbf{u}$ with $r > 0$.
 b) True, as the dot product between these two vectors is zero.
 c) False, as \mathbf{w} is parallel to the x-axis when $\mathbf{u} = (1, 0, 0)$ and $\mathbf{v} = (-1, 0, 0)$.

4. a) The line intersects the plane at the point $\frac{1}{21}(267, -40, 2)$.
 b) The distance from the origin to Π is 9.
 c) $-\frac{28}{3}$.

5. In Cartesian coordinates, $r = 2 \cos \theta$ reads $\sqrt{x^2 + y^2} = 2x / \sqrt{x^2 + y^2}$. Multiply through by $\sqrt{x^2 + y^2}$ to find that $x^2 + y^2 - 2x = 0$. Add 1 to each side to find that $x^2 - 2x + 1 + y^2 = 1$ which is to say, $(x - 1)^2 + y^2 = 1$. Thus, the circle has its center at the point $(1, 0)$ and its radius is 1.

6. a) If \mathbf{u} is tangent to Π , then \mathbf{u} is perpendicular to a normal vector to Π . In our case, the vector $\mathbf{n} = (1, 1, 1)$ is a normal vector to Π since it is perpendicular to both the vector that points from A to B (which is $(-1, 1, 0)$) and the vector, $(-1, 0, 1)$, which points from A to C. Meanwhile, $\mathbf{u} \cdot \mathbf{n} = 1 + 2 - 3 = 0$.
 b) $\mathbf{v} = \mathbf{n} \times \mathbf{u} = (-5, 4, 1)$ is such a vector.
 c) The vector $\mathbf{w} - |\mathbf{u}|^{-2} (\mathbf{w} \cdot \mathbf{u}) \mathbf{u}$ is orthogonal to \mathbf{n} since both \mathbf{w} and \mathbf{u} are. Also, it is orthogonal to \mathbf{u} since its dot product with \mathbf{u} is zero. Thus, it must be a multiple of \mathbf{v} since \mathbf{v} is also orthogonal to both \mathbf{u} and \mathbf{n} . You can think of \mathbf{n} and \mathbf{u} as pointing along two orthogonal axis in space and then \mathbf{v} , being orthogonal to both \mathbf{n} and \mathbf{u} ,

lies along the third axis. As \mathbf{w} has zero dot product with both \mathbf{n} and \mathbf{u} , like \mathbf{v} , it is parallel to the third axis and so is a multiple of \mathbf{v} .