Math 21a Hourly 2 Answers

1. a) $\nabla f = (3, 4)$.
   b) The tangent plane is given by $z - 3x - 4y = -5$.
   c) Using the linear approximation, $f(1.02, 2.05) = f(1, 2) + \nabla f|_{(1,2)} \bullet (.02, .05) = 6.26$.

2. The stationary points occur at $(e^{-a}, e^{-b}, e^{-c})/(e^{-a} + e^{-b} + e^{-c})$.

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4. a) The stationary points are (0, 1), (0, -1), (1, 1), (1, -1), (-1, 1), (-1, -1).
   b) The local maximum is (0, -1). The local minima are (1, 1), (-1, 1). The remaining three are saddles.
   c) If the level set is tangent to the y axis, then ∇f is orthogonal to (0, 1) and so $f_y = 0$. This occurs where $y = \pm 1$. Where $y = 1$, $f = x^4 - 2x^2 - 6$ and so $f = 2$ if $x = \pm 2$. Where $y = -1$, $f = x^4 - 2x^2 + 6$ and so f is not equal to 2 for any value of x. Thus, the points are (2, 1) and (-2, 1).

5. Change the order of integration to write this integral as $\int_0^{\pi/2}\left(\int_{\sin(y)}^1 dx\right)dy = \pi/2 - 1$.

6. Let z denote height, x denote length and y denote width. The you are asked to minimize the function $f(x, y, z) = 50\,x\,y + 20\,(x\,z + y\,z)$ where x, y and z are constrained by the requirement $x\,y\,z = 20{,}000{,}000$. The minimum has z = 500 centimeters and x = y = 200 centimeters.