

Math 21a Practice Hourly 1 Answers
(Fall 2000)

Note: Problems 1-6 count 8 points each, while Problem 7 counts for 2 points.

1. Answer:

- a) $|\mathbf{v}| = \sqrt{21}$, $|\mathbf{w}| = 1$, $|\mathbf{v} - 3\mathbf{w}| = \sqrt{6}$.
- b) This number is $\mathbf{v} \cdot \mathbf{w} = 4$.
- c) $(-2, 0, 1)$.
- d) $b = -2$ and $c = 0$.

2. Answer:

- a) $(3/2, 0, 0)$, $(0, -3/2, 0)$ and $(0, 0, 3)$. (There are infinitely many other possibilities)
- b) $(2, -2, 1)$, or any non-zero multiple
- c) The distance is 1.
- d) Parametric: Send $t \rightarrow (3t, 3t, 3)$. Nonparametric: $x = y$ & $z = 3 - 2x + 2y$.
(There are infinitely many other possibilities.)

3. Answer:

- a) $(0, -3, 4\pi)$.
- b) $2\sqrt{\pi}(-3, 0, 4)$
- c) $t \rightarrow (-3t, -3, 4(\pi + t))$.
- d) The velocity vector at general t is $2t(3\cos(t^2), -3\sin(t^2), 4)$ whose length is $10t$.
Thus, the distance is the integral of this last function from 0 to $\sqrt{\pi}$ which is $5\sqrt{\pi}$.

4. Answer:

- a) \mathbf{p} is on L and \mathbf{v} is tangent to L .
- b) $t \rightarrow \mathbf{p} + t\mathbf{v}$.
- c) $(-4, -3, 0)$; the case of $t = -1$ in the preceding parameterization.
- d) $d = |\mathbf{p} \times \mathbf{v}|/|\mathbf{v}| = \sqrt{\frac{34}{35}}$.

5. Answer:

- a) The plane where $x - 3z = 0$. (There are infinitely many other possibilities.)
- b) $\mathbf{w} \cdot \mathbf{v} = 0$ requires $b = 16$.
- c) $b = -6$ and $c = -2$.
- d) If \mathbf{s} is parallel to \mathbf{v} , then $\mathbf{u} \times \mathbf{s}$ will be perpendicular to \mathbf{v} . For this, take $c = -6$.

6. Answer:

- a) True. Indeed, since $\mathbf{r} \cdot \mathbf{r} = |\mathbf{r}|^2$ and $\frac{d}{dt}(\mathbf{r} \cdot \mathbf{r}) = 2 \mathbf{r} \cdot \frac{d}{dt} \mathbf{r}$, which is zero, the distance of the particle to the origin stays constant. Thus, it moves on the surface of a sphere.
- b) False: If the particle motion is given by $\mathbf{r}(t) = t \mathbf{k}$, then it moves on a line and not a sphere.
- c) True: See the preceding answer. In fact, any $\mathbf{r}(t)$ of the form $\mathbf{r}(t) = \mathbf{r}_0 + t \mathbf{k}$ will do.
- d) True: Write $\mathbf{r}(t) = (a(t), b(t), c(t))$, so $\mathbf{k} \times \mathbf{r} = (-b(t), a(t), 0)$ and $\mathbf{k} \times \frac{d}{dt} \mathbf{r} = (-\frac{d}{dt} b(t), \frac{d}{dt} a(t), 0)$. As this is orthogonal to $\mathbf{k} \times \mathbf{r}$, so $a \frac{d}{dt} a + b \frac{d}{dt} b = 0$, which says that $2^{-1} \frac{d}{dt} (a^2 + b^2) = 0$ so $a^2 + b^2$ is constant. Thus, the x and y coordinates of the particle move on a circle while the z coordinate can do what it likes. This puts the motion on a circular cylinder.

7. Answer: 0. The vector $\mathbf{u} \times \mathbf{v}$ is perpendicular to both \mathbf{u} and \mathbf{v} .