

13. a) In the first case, the plane is where  $z = 1$ . In the second, it is where  $x + z = 1$ .  
 b) In the first case, the plane is where  $2x - y = 2$ . In the second, it is where  $x + y - z = 1$ .
14.  $L = 2x - 5y + z$ .
15. a) 2. b) 1. c)  $-\sqrt{2}$ .
16.  $\nabla g = (-2, -4)$ .
17. a)  $\nabla f = (-\sin(x), 2y)$  so possible the critical points have the form  $(n\pi, 0)$  with  $n$  an integer.  
 The second derivative tests finds  $(n\pi, 0)$  a local minimum when  $n$  is odd and a saddle when  $n$  is even.  
 b)  $\nabla f = (-\sin(x)\sin(y), \cos(x)\cos(y))$  so the critical points have the form  $(n\pi, (m + 1/2)\pi)$  and  $((n + 1/2)\pi, m\pi)$  where  $n$  and  $m$  are integers. In the first case, if both  $n$  and  $m$  are odd or both are even, it is a local maximum. If one is odd and the other not, then it is a local minimum. In the second case, it is a saddle for all  $n$  and  $m$ . (Use the 2<sup>nd</sup> derivative test.)  
 c)  $\nabla f = (2x, 3y^2 - 3)$  so the critical points are  $(0, \pm 1)$ . The point  $(0, 1)$  is a local minimum and the point  $(0, -1)$  is a local minimum.
18. The origin is the only interior critical point and it is a saddle. The extreme points on the boundary occur at  $(\pm 3/\sqrt{2}, \pm\sqrt{2})$ . The points  $(3/\sqrt{2}, \sqrt{2})$  and  $(-3/\sqrt{2}, -\sqrt{2})$  are maxima and the others are minima.
19. The maxima occur at  $(\pm 1, 0, 0)$  and the minima at  $(0, \pm 1, 0)$ .
20. The maximum is at  $(1, 0, 0)$ .
21. The closest points are  $(0, \pm 1, \pm 1)$  with distance  $\sqrt{2}$ .
22. The square of the length of  $\mathbf{E}$  is  $x^4 + 4y^2 + z^2 + z^2y^2 - 2zy + 1$  which is smallest at the origin.
23.  $3(\pi/2 - 1)$ .
24.  $4/(3\pi)$ . (Remember to divide the integral of  $x$  by the area.)
25.  $\pi/8$ .
26.  $\sin(1)/2$ .