27. $\pi/2$

28. $\pi \sin(1)$.

29. $4\pi (1 - \cos(1))/3$.

30. $\int_0^1 \left(\int_0^r \left(\int_0^\pi \left(r^3 \sin(\theta)\cos^2(\theta) \right) d\theta \right) dz \right) r\,dr = 1/9.$

31. $\int_0^{\pi/2} \left(\int_0^{\pi/4} \left(\int_0^{1/\cos(\varphi)} \left(\rho^4 \sin(\theta)\cos(\theta)\sin^2(\phi)\cos^2(\phi) \right) \rho^2 d\rho \right) \sin(\phi) d\phi \right) d\theta = 1/56.$

32. $\int_{-3/5}^{3/5} \left(\int_{-\sqrt{9/25-x^2}}^{\sqrt{9/25-x^2}} \left(\int_{\frac{4}{3}\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz \right) dy \right) dx =$

$\int_0^{4/5} \left(\int_{\frac{3}{4}r}^{\sqrt{1-r^2}} \left(\int_0^{2\pi} d\theta \right) dz \right) r\,dr =$

$\int_0^1 \left(\int_0^{\mathrm{Arc}\tan(4/3)} \left(\int_0^{2\pi} d\theta \right) \sin(\phi) d\phi \right) \rho^2 d\rho = 4\pi/15.$

33. a) $\int_0^\infty \int_0^{2\pi} e^{-r^2} d\theta \, r\,dr = \pi$.

b) The square of this integral is the Cartesian coordinate version of the preceding integral, so the integral in question equals $\pi^{1/2}$.

34. $u(t, x) = e^t (1 + (x - 4t)^2)^{-1}$.

35. a) Advection. b) Advection. c) Diffusion.

36. 0

37. $-2/\pi - 2/7$

38. 0.

39. a) $\mathbf{X}(u, v) = (2 (1 - u^2/9 - v^2/25)^{1/2}, u, v)$ for values of (u, v) with $u^2/9 + v^2/25 \le 1$.

b) $\mathbf{X}(u, v) = (u, 3 (1 - u^2/4 - v^2/25)^{1/2}, v)$ for values of (u, v) with $u^2/4 + v^2/25 \le 1$.

c) $\mathbf{X}(u, v) = (1 - u^2 - v^4, u, v)$ for values of (u, v) with $u^2 + v^4 \le 1$.

d) $\mathbf{X}(u, v) = (u, v, 2 - u - v)$ for values of (u, v) in $u \ge 0$, $v \ge 0$ and $u + v \le 2$.