40. a) $\int_{-5}^{5}\left(\int_{-3\sqrt{1-v^2/25}}^{3\sqrt{1-v^2/25}} \sqrt{1+4(u^2/81+v^2/625)/(1-u^2/9-v^2/25)}\,du\right)dv$.

b) $\int_{-5}^{5}\left(\int_{-2\sqrt{1-v^2/25}}^{2\sqrt{1-v^2/25}} \sqrt{1+9(u^2/16+v^2/625)/(1-u^2/4-v^2/25)}\,du\right)dv$.

c) $\int_{-1}^{1}\left(\int_{-\sqrt{1-v^4}}^{\sqrt{1-v^4}} \sqrt{1+4u^2+16v^6}\,du\right)dv$.

41. 10/3. (The area of the surface is π and the integral of z over the surface is $10\pi/3$.)

42. sin(1).

43. $500\,\pi/3$.

44. (x, -y, 0).

45. a) $(x^2yz/2, 0, 0)$.

b) (2z, 3x, y).

46. a) $(0, x/\pi)$.

b) (x, 0, 0).

47. a) No such vector field exists because (x, -2y, xy) has divergence -1 and the divergence of a curl is zero.

b) $(xy\cos(yz^2), 0, 0)$.

48. 5x + 3y = 0.

49. $\iint_S x\,g\,dS$. Let $\mathbf{E} = (g, 0, 0)$; here $\mathrm{div}(\mathbf{E}) = f$ while $\mathbf{E}\bullet\mathbf{n} = x\,g$ on the surface of the volume, V, in question. Thus, the divergence theorem implies that $\iint_S x\,g\,dS = \iiint_V f\,dV$.

50. a) 2π. This is a direct computation: Parameterize the circle by $t \to (\cos(t), \sin(t))$ and then path integral is just the integral of dt between 0 and 2π.

b) 2π. Use Green's theorem for a region with holes and note that the vector field in question has zero 'curl' in the sense that when written as (P, Q), then $Q_x - P_y = 0$.

c) 0. This is also a direct application of Green's theorem.

51. A = (-1, 0, 0, 0, 3), B = (-1, -1, 0, 2, 2).

52. The mean is $m = (n_1 m_1 + n_2 m_2)/(n_1 + n_2)$ and the standard deviation is $s = ((n_1 - 1)\,s_1^2 + (n_2 - 1)\,s_2^2)^{1/2}/(n_1 + n_2 - 1)^{1/2}$.