

## Suggested problems for the Differential Equation lectures

### Suggested problems for the 12/11 lecture

- Find the general form for the solution,  $p(t)$ , to the following equations:
  - $p' = c p^2$  where  $c$  is a constant.
  - $p' = c p^m$  where  $c$  is a constant and  $m$  is an integer.
- The equation  $p' = p(1 - p)$  is known as a 'logistics equation'. Its general solution can be found by the method that is described in Section 1c of the Differential Equation supplement.
  - Verify by plugging into the logistic equation that  $p(t) = a e^t / (1 + a e^t)$  is a solution for any constant  $a$ .
  - Prove that there is a solution of the logistics equation whose time  $t = 0$  value can be any given real number. That is, given any real number  $p_0$ , find a solution  $p(t)$  to the logistics equation with  $p(0) = p_0$ .
  - Write down the version of Equation (2.18) of the Differential Equation supplement in the case where  $f(x) = x(1 - x)$  comes from the logistics equation and where  $x_0 = 0$ . Then, write down the solution,  $x(t)$ , to the resulting equation that satisfies  $x(0) = .001$ .
  - Use  $p(t)$  to denote the logistics equation solution that has  $p(0) = .001$ ; and let  $x(t)$  be your solution from the Part c of this problem. Give a rough estimate for  $|p(t) - x(t)|$  when  $t < 1$ .

### Suggested problems for the 12/13 lecture

- Find the solutions to the equation

$$\frac{\partial}{\partial t} u(t, x) = -2 \frac{\partial}{\partial x} u(t, x) + u(t, x)$$

which obey the following:

- $u(0, x) = \cos(x)$ .
  - $u(0, x) = \sin(x)$ .
  - $u(1, x) = \cos(x)$ .
  - $u(2, x) = \cos(x)$ .
- Verify by taking the appropriate derivatives that the function in Equation (4.19) of the Differential Equation supplement obeys Equation (4.7) in the same supplement.

3. First, give a heuristic argument for the claim that any equation that is purported to model the time and or space evolution of the population of non-interacting entities should obey the superposition principle: *If  $u_1$  and  $u_2$  are solutions to the equation in question and if  $a_1$  and  $a_2$  are constants, then  $u = a_1 u_1 + a_2 u_2$  is also a solution to the given equation.* Next, having made your argument, reread Section 1d of the Differential Equation supplement and consider the exponential growth equation  $p' = a p$  with  $a =$  constant. Do you expect this equation to obey the superposition principle? Does it?

### Suggested problems for the 12/15 lecture

1. Use (5.7) in the Differential Equation supplement to find a solution,  $u(x)$ , to the equation  $-u_{xx} = k$  in the case where  $k(x)$  is given by

$$k(x) = \begin{cases} 0 & \text{when } x < 0 \\ e^{-x} & \text{when } x \geq 0 \end{cases}.$$

2. Fix a vector  $\vec{s}$  in  $\mathbb{R}^3$  and then consider  $G = \frac{1}{4\pi} \frac{1}{|\vec{r} - \vec{s}|}$  as a function of  $\vec{r} = (x, y, z)$ . Take the appropriate derivatives to verify that  $G_{xx} + G_{yy} + G_{zz} = 0$  where  $\vec{r} \neq \vec{s}$ .