Answers to Math 21a Hourly 1

(Fall 2001)

1. a) F b) T c) T d) T e) T f) F

2. a) T or F depending on your interpretation. b) F c) T
   d) $2x + y - 3z = 2$. Alternately, you can $\Pi = (1, 0, 0) + t\,\mathbf{a} + s\,\mathbf{b}$ where t and s are real numbers and **a** and **b** are non-zero vectors that have zero dot product with **n**.

   Note that a)-c) counted 2 points each while d) counted for 4 points.

3. a) $\mathbf{q}'(t) = \frac{1}{\sqrt{2}}\,(-\sin(t)\,e^t + \cos(t)\,e^t,\ \cos(t)\,e^t + \sin(t)\,e^t)$.

   b) As $|\mathbf{q}'(t)| = e^t$, the length of the path is $\int_0^1 e^t dt = e - 1$.

4. The answer is $t = (\frac{1}{2} + n)\,\pi$, where n is any integer.
   Solution 1: Find that $y'(t) = 2\cos(t)(\sin(t) + 4)$, and so vanishes only where cos(t) does. In this regard, $x(t) = (8 + \sin(t))\cos(t)$ and $y(t) = (8 + \sin(t))\sin(t)$.
   Solution 2: Draw the graph.

5. a) If Q=(1,0,0), P=(0,0,0), then P-Q is in the plane as well as v=(4,2,-4).
   The vector n=(P-Q) x v=(0,4,2) is normal to the plane so that 4y+2y=d is the equation of the plane. Because (0,0,0) is on the plane, 4y+2y=0.
   b) Use the formula |v x (P-Q)|/|v|=sqrt(17)/3
   c) Use the formula |n . (P-Q)|/|n|=3/sqrt{5}

6. a) (-10, -2, 500)
   b) Coyote hits the ground at t = 0. At this point, Coyote's coordinates are (0, -2, 0) while Road Runner's are (0, 0, 0). Thus, Coyote is at distance 2 from Road Runner.
   c) Coyote's velocity vector is (1, 0, -100 – 10 t), so equals (1, 0, -100) at t = 0 when the ground is hit. His speed is the length of this vector, $(10{,}001)^{1/2}$.
   d) The cosine of the angle is the ratio of the dot product of the velocity vector with the unit vector (0, 0, 1). Thus, the cosine is $-100\,(10{,}001)^{-1/2}$.