# Math 21a Review Problems

- The problems given below sample the material from the course which you will be responsible for on the final examination.
- The level of difficulty of these problems should roughly correspond to the average level of difficulty of those which will appear on the exam. Of course, there may be problems on the final exam which are somewhat longer or more involved.
- Since electronic aids will not be allowed in the exam room, use these aids in your review only as a last resort.
- The answers to the problems below appear at the end.
- Problems 1-35 are relevant for all sections of Math 21a.
- Problems 36-50 are relevant only for students in the Regular and Physics sections. They are not meant for students in the BioChem or Computer Science section.
- Problems 50-62 are meant for the students in the BioChem sections and are not meant for those in the Regular, Physics or Computer Science sections.
- Problems 63-77 are meant for the students in the Computer Science section and are not meant for those in the Regular, Physics or BioChem sections.
- Students can obtain additional answered review problems by working the relevant odd numbered problems from Chapters 9.4-9.9, 10.1-10.5, 10.7, 11.1, 11.3, 12.1-12.10, 13.1-13.6, 14.1-14.8 and at the ends of Chapters 9-14 in Part II of the 9'th edition of Thomas and Finney's book Calculus. The latter is on reserve in Cabot Library. Moreover, almost any book on multi-variable calculus will cover essentially the same subjects as we did here. Thus, even more problems for review can be had by working answered problems in other multivariable calculus books. (Please don't take such books out of Cabot; zerox some problems instead so that other students can have access to the same resource.) BioChem section students who wish to work additional problems in Probability and Statistics should work more of the answered problems from Rosner's book. Meanwhile, students in the Computer Science section can work more of the answered problems from Apostol's book.

**PROBLEMS:**

1. Give an equation of the form $f(x, y) = 0$ for the following parametrized curves in $R^2$:
   a) $x = (t^2 + 1)^{1/4}$, $y = 1 - t$.
   b) $x = 2 \tan(t)$, $y = 1/\cos(t)$ for $-\pi/2 < t < \pi/2$.
   c) $x = 4 \cos(t)$, $y = 3 \sin(t)$.

2. In each case, give an equation for the line in $R^2$ which is tangent to the given curve at the indicated point:
   a) The curve is parametrized as $x = (t^2 + 1)^{1/4}$, $y = t$ and the point is where $t = 0$.

b) The curve is where $x^3 + y^2 = -23$ and the point is (-3, 2).

c) The curve is where $x + y^3 = 1$ and the point is (2, -1).

3. Find the length of the curve parametrized by $x = e^{2t} - 2t$, $y = 4\,e^t$ for $0 \le t \le 1$.

4. a) Find a parametrization of the form $t \to (x(t), y(t))$ for the curve in $R^2$ which is parameterized in polar coordinates by $r = t$, $\theta = t^3$ with $t \ge 0$.

   b) Write this curve in the form $f(x, y) = 0$.

5. In each of the cases below, write the vector **B** as a sum of a vector which is parallel to the vector **A** and which is perpendicular to **A**.

   a) **A** = (1, 2, 2) and **B** = (1, 2, -1).

   b) **A** = (3, -4, 0) and **B** = (5, 1, 1).

   c) **A** = (2, -1, -2) and **B** = (3, 3, 3).

6. Suppose that **v** and **w** are vectors in $R^3$ with $|\mathbf{v}| = 2$ and $|\mathbf{w}| = 3$. In each case below $|2\mathbf{v} - \mathbf{w}|$ is given. Decide whether **v** and **w** are perpendicular or not, or whether there is not enough information to decide.

   a) $|2\mathbf{v} - \mathbf{w}| = 12$.

   b) $|2\mathbf{v} - \mathbf{w}| = 7$.

   c) $|2\mathbf{v} - \mathbf{w}| = 5$.

7. Find the distance from the point (1, 2, 1) to the following:

   a) The plane where $2\,x + y - 2\,z = 0$.

   b) The line parameterized by $t \to (6\,t, 3\,t + 2, 2\,t + 1)$.

8. In each case, find an equation of the form $f(x, y, z) = 0$ for the indicated plane:

   a) The plane through the point (1, 0, 0) which is normal to **A** = (2, 1, -1).

   b) The plane containing the points (1, 1, -1), (2, 1, 0) and (3, 3, 3).

   c) The plane through the point (-1, 0, 0) for which **A** = (-1, -1, 1) and **B** = (1, 1, 3) are tangent.

9. Find the absolute value of the cosine of the angle between the planes $x = 5$ and $6\,x + 3\,y + 2\,z = 2$.

10. Write a parametric equation for the line through the origin which is normal to the plane through the three points (0, 1, 0), (1, -1, 1) and (1, 1, -1).

11. In each of the cases below, the given vector function of the parameter t is meant to be the velocity vector of a parametrized curve in $R^3$. Decide whether the given curve lies entirely in a single plane.

    a) $\mathbf{v}(t) = (5 \cos(t), 3 \sin(t), \cos(t))$.
    b) $\mathbf{v}(t) = (8 \, t^2, 3 \, t, \cos(t))$.
    c) $\mathbf{v}(t) = (8 \, t^2, \cos(t), -7 \cos(t))$.

12. In each case, find the linear approximations to the given function at the indicated points:

    a) $f(x, y, z) = 10 \, x^2 + y \, z - z^2 + 1$;  and the points are (1, 1, 1) and (1, 1, -1).
    b) $f(x, y, z) = \sin(xyz^2) + z$; and the points are (1, 2, 0) and (3, 0, 1).

13. In each case, find the equation for the tangent plane to the given surface at the indicated points:

    a) The surface is where $e^{xyz} - 2 + z = 0$ and the points are (0, 0, 1) and (0, 1, 1).
    b) Thus surface is where $x^2 + y^2 - xyz = 1$ and the points are (1, 0, 1) and (1, 1, 1).

14. Write down the linear approximation at (1, 1, 1) for any function f(x, y, z) on $R^3$ with following properties:

    a) $f(1, 1, 1) = -2$
    b) Both $\mathbf{A} = (1, 1, 3)$ and $\mathbf{B} = (3, 1, -1)$ are tangent to the level set f = -2 at (1, 1, 1).
    c) The directional derivative of f at (1, 1, 1) in the direction (1, 0, 0) is 2.

15. Let $f(x, y, z) = x^2 - yz + 3$.  In each case below, the given point lies on a parametrized curve and the given vector $\mathbf{v}$ is the tangent vector to that curve at the given point.  Give the instantaneous rate of change of f along the curve at the given point.

    a) The point is (1, 1, 1) and $\mathbf{v} = (1, 0, 0)$.
    b) The point is (1, -1, 1) and $\mathbf{v} = (0, -1, 0)$.
    c) The point is (0, 2, 2) and $\mathbf{v} = (1, 1, 0)$.

16. Suppose that f(x, y) is a function on $R^2$ whose gradient at the origin in (1, -3).  Also, suppose that (x(u, v), y(u, v)) is a function from $R^2$ to $R^2$ which sends (0, 0) to (0, 0).  Also, suppose the partial derivatives of x(u, v) and y(u, v) at (0, 0) are: $x_u = 1$, $y_u = 1$, $x_v = -1$, $y_v = 1$.  Give the gradient vector at (0, 0) of the function g(u, v) = f(x(u, v), y(u, v)).

17. In each case, give the local maxima, then the local minima, and finally, the saddle points for the given function:

    a) $f(x, y) = y^2 + \cos(x)$ .
    b) $f(x, y) = \cos(x) \sin(y)$ .
    c) $f(x, y) = y^3 - 3y + x^2$.

18. Find the points where the function $f(x, y) = 3xy + 1$ takes on its maximum and minimum values on the region where $x^2/9 + y^2/4 \le 1$.

19. Find the points of the ball $x^2 + y^2 + z^2 \le 1$ where the function $f(x, y, z) = 2 x^2 - y^2 - \cos(z^2)$ achieves its maximum and minimum values.

20. Find the point or points on the surface $x + y^2 + z^2 = 1$ where $f(x, y, z) = x/(1 + y^2 + z^2)$ has its maximum.

21. Find the distance to the origin of the point on the surface where $x^2/4 + z^2y^2 = 1$ which is closest to the origin.

22. Let $\mathbf{E}(x, y, z) = (x^2, zy + 1, 2y - z)$. Find the point or points in $R^3$ where the length of $\mathbf{E}$ is least.

23. Find the area of the region where $x^2/4 + y^2/9 \le 1$ and $x/2 + y/3 \ge 1$.

24. Find the average value of $f(x, y) = x$ on the part of the first quadrant where $x^2 + y^2 \le 1$.

25. Find the area of the region given in polar coordinates by $0 \le r \le \cos(\theta)$ and $0 \le \theta \le \pi/2$.

26. Integrate the function $f(x, y) = \cos(x^2)$ over the region in the first quadrant where $x - y \ge 0$ and $x \le 1$.

27. Find the volume of the region in $R^3$ where $y \ge 0$ and $y + x^2 + z^2 \le 1$.

28. Find the volume of the region in $R^3$ where $0 \le z \le \cos(x^2 + y^2)$ and $x^2 + y^2 \le 1$.

29. Integrate the function $f(x, y, z) = \sin((x^2 + y^2 + z^2)^{3/2})$ over the interior of the ball of radius 1 in $R^3$ whose center is at the origin.

30. Write in cylindrical coordinates the integral of the function $f(x, y, z) = y x^2$ over the region where $-1 \le x \le 1, 0 \le y \le (1 - x^2)^{1/2}$ and $0 \le z \le (x^2 + y^2)^{1/2}$. Then, evaluate the resulting integral.

31. Write in spherical coordinates the integral of the function $xyz^2$ over the portion of the first quadrant where $(x^2 + y^2)^{1/2} \le z \le 1$. Then, evaluate the resulting integral.

32. Consider the inverted cone whose vertex lies at the origin and whose base is the disc in the plane $z = 3$ with center $(0, 0, 3)$ and radius 4. Express the volume of the portion of this cone

where $x^2 + y^2 + z^2 \leq 1$ as an iterated integral in rectangular, cylindrical and also spherical coordinates. Finally, find this volume by evaluating one of your integral expressions.

33. a) Write the integral $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \, dx \, dy$ in polar coordinates and evaluate.

b) Evaluate $\int_{-\infty}^{\infty} e^{-x^2} \, dx$. (Hint: Consider the square of this integral.)

34. Write down the function u(t, x) which solves the equation $u_t = -4 u_x + u$ and equals $(1 + x^2)^{-1}$ at t = 0.

35. A typical advection equation for a function u(t, x) has the form $u_t = -c u_x + k$, where c is a constant and k is a function of t and x which might depend on u. On the other hand, a typical diffusion equation for a function u(t, x) has the form $u_t = \mu u_{xx} + k$, where μ is a constant and where k is a function which might depend on u. In each of the following cases, indicate whether the function in question is more likely to obey an advection or a diffusion equation:
   a) The function u(t, x) gives the density of a heart stimulating drug in the blood as a function of time t and distance x along a vein leading to the heart. The drug is injected in the vein at t = 0.
   b) The function u(t, x) gives the density of cars on the west bound side of the Massachusetts Turnpike as a function of time t and distance x from the Boston end.
   c) The function u(t, x) gives the temperature of glacial ice in Greenland as a function of time t and distance x from the surface of the glacier.


- **The following review problems are only for the Regular and Physics sections.**

36. Compute the line integral of $\mathbf{F} = (y \sin(\pi z/4), x \cos(\pi z/4), - (y^2 + x^2) z^4)$ around the circle which is cut from the sphere $x^2 + y^2 + z^2 = 10$ by the plane z = -3. Traverse this circle counter-clockwise as viewed from the z = 0 plane.

37. Compute the line integral of the vector field $\mathbf{F} = (-\sin(\pi z), \cos(\pi z), - (y^2 + x^2) z^4)$ along the straight line segment starting at the origin and ending at (1, 1, 1).

38. Compute the line integral of the vector field $\mathbf{F} = (x y^4 \sin(\pi y^5/2), yx^2)$ along the path which traverses (counter-clockwise) the square in the x-y plane with vertices (0, 0), (1, 0), (0, 1) and (1, 1).

39. Parametrize the following surfaces:
   a) The portion of the surface where $x^2/4 + y^2/9 + z^2/25 = 1$ which lies where $x \geq 0$.

b) The portion of the same surface which lies where $y \geq 0$.

c) The portion of the surface where $x + y^2 + z^4 = 1$ which lies where $x \geq 0$.

d) The portion of the plane where $x + y + z = 2$ which lies where x, y and z are all positive.

40. For Cases a-c in the previous problem, write down an iterated integral which computes the area of the indicated portion of the surface.

41. Find the average height over the x-y plane of the $z \geq 0$ part of the surface $x^2 + y^2 + z^2/25 = 1$.

42. Find the flux of $\mathbf{F} = (\sin(x^4 + z^3), \cos(x + z(y^2 - 1), z\,x^2)$ through the part of the $y = 1$ plane where both z and x are positive, but both are less than 1. Use the normal which has positive dot product with the position vector at the point (0, 1, 0).

43. Find the flux of $\mathbf{F} = (0, 0, z)$ through the surface $x^2 + y^2 + z^2 = 25$. Use the normal which points away from the origin.

44. Write down a vector field on $R^3$ which is not constant, but has zero curl and zero divergence.

45. a) Write down a vector field on $R^3$ which has divergence equal to xyz.
    b) Write down a vector field on $R^3$ whose curl is equal to (1, 2, 3).

46. a) Write down a vector field on the plane whose counter-clockwise path integral around any circle equals the square of the radius of the circle.
    b) Write down a vector field on $R^3$ whose outward flux through the surface of any cube is equal to the third power of the length of an edge.

47. a) Either exhibit a vector field whose curl is (x, -2y, xy), or explain why no such vector field can exist.
    b) Either exhibit a vector field whose divergence is $y \cos(y\,z^2)$, or explain why no such vector field can exist.

48. Suppose $\mathbf{F}$ is a vector field whose curl is equal to (3, -5, 7). For each plane through the origin, one can consider the path integral of $\mathbf{F}$ in either direction around the circle where the plane intersects the sphere where $x^2 + y^2 + z^2 = 4$. Give the equation of a such a plane for which the path integral in question is zero.

49. Let f(x, y, z) be some (unspecified) function and let $g(x, y, z) = \int_0^x f(s, y, z)\,ds$. Give a formula which expresses the integral of the function f over the ball where $x^2 + y^2 + z^2 \leq 1$ as a surface

integral over the boundary sphere which involves the function g.  Use the divergence theorem to justify the validity of your formula.

50. In this problem, $\mathbf{F} = (-y, x)/(x^2 + y^2)$.  This is a vector field which is defined everywhere on the plane except at the origin.
    a)  Compute the counter-clockwise path integral of $\mathbf{F}$ around the circle $x^2 + y^2 = 1$.
    b)  Suppose that $\gamma$ is a closed loop in the plane which encloses a region that contains the disk where $x^2 + y^2 \le 1$.  Compute the counter-clockwise path integral of $\mathbf{F}$ around $\gamma$.
    c)  Suppose now that $\gamma$ is a closed loop which encloses a region which does not contain the origin.  Compute the counter-clockwise path integral of $\mathbf{F}$ around $\gamma$.
    In all of the above, justify your answers.

- **The following problems are only for students in the BioChem sections**

51.  In each case below, give two distinct sets, $A = (a_1, a_2, a_3, a_4, a_5)$ and $B = (b_1, b_2, b_3, b_4, b_5)$ of integers (negative, zero or positive) for which the corresponding means, medians and standard deviations have the stated relationship:
    a)  mean(A) = mean(B)  and median(A) = median(B).
    b)  mean(A) = mean(B) and standard deviation(A) = standard deviation(B).
    c)  mean(A) = mean(B), median(A) = median(B), standard deviation(A) = standard deviation(B).

52.  Suppose two data sets of size $n_1$ and $n_2$ have respective means $m_1$ and $m_2$, and also respective standard deviations $s_1$ and $s_2$.  Pool these data sets to obtain a third, with size $n_1 + n_2$.  Give formula for the mean and standard deviation of this pooled data set in terms of $m_1$, $m_2$, $s_1$, $s_2$.

(You may use a calculator or tables in the Rosner text to do computations for the following problems.  Similar problems on the exam will be worded so that calculators are not required.)

53.  Answer Problems 3.18-3.23 on page 67 of the Rosner text.

54.  Answer Problems 3.85, 3.87 and 3.88 on page 72 of the Rosner text.

55.  Answer Problems 3.104 and 3.105 on pages 73-74 of the Rosner text.

56.  Answer Problem 4.37 on page 110 of the Rosner text.

57.  Answer Problems 4.44 on page 110 of the Rosner text.

58. Answer Problems 4.69 and 4.70 on pages 113-114 of the Rosner text.

59. Answer Problem 4.78 on page 114 of the Rosner text.

60. Answer Problems 5.31 and 5.32 on pages 148-149 of the Rosner text.

61. Answer Problems 5.61 and 5.62 on page 151 of the Rosner text.

62. Answer Problems 5.64 and 5.65 of the Rosner text.

- **The following problems are only for students in the Computer Science section.**

63. Let S be the set of 10,000 undergraduates at a university. Here are some subsets:

   A is the set of 2,500 freshmen.
   B is the set of 3,000 atheletes.
   C is the set of 5,000 females.
   D is the set of 200 football players.

   There are 1,200 female atheletes but none play football. Of the freshmen, 1,000 are atheletes
   and 1,250 are women. Note that 400 are both women and atheletes.
   a) Express each of the following subsets in terms of A, B and C; then specify its size.
      1) The set of all females that are not freshmen.
      2) The set of all male freshmen that are not atheletes.
   b) Translate into the language of sets the following statement: "A football playing freshman is
      a male athelete."
   c) An alumnus asks the university to choose a student at random to receive a scholarship.
      What is the probability that the recipient is either a male freshman athlete or a non-freshman
      women?

64. a) Determine how many ways there are to select a subset of 5 of the 13 spades in a standard
       deck of 52 cards. You may leave your answer in terms of products of integers but expand
       any factorials or binomial coefficients.
    b) Determine how many distinct bridge hands contain 5 spades, 3 hearts, 3 diamonds and 2
       clubs. As above, you may leave your answer in terms of products of integers but expand
       any factorials or binomial coefficients.
    c) Determine how many distinct bridge hands have 5 cards in the longest suit, 2 cards in the
       shortest suit and 3 cards in each of the remaining two suits.

65. The Math 21a Course Head, anticipating the need for make-up midterm exam, has written two

easy exams and two hard exams. The easy exams are the same, but the hard ones are not; though equally hard, H1 has one less question than H2. The Course Head puts the exams randomly into four numbered envelopes. Subsequently, a student arrives for the exam and is invited to select one of the four envelopes at random. The student chooses envelope #1 but does note open it. The Course Head then says: "I thought you might want to see what one of my hard exams looks like. Let me show you the shortest one from the remaining envelopes." The Course Head then shows the exam from envelope #2 to the student who visibly blanches. The Course Head then asks: "Would you like to exchange your envelope before you open it for one of the remaining two envelopes, #3 or #4?" Let A denote the event that envelope #1 contains a hard exam and let B denote the event that envelope 2 contains the shortest of the hard exams in envelopes #2-4.

   a) Enumerate all of the ways that can lead to event B by specifying the distribution of the exams, E, E, H1 and H2 between the various envelopes; then assign a probability to each.

   b) Calculate the conditional probability of A occurring given B.

   c) If the student does not make the exchange, what is the probability of the student getting a hard exam?

   d) If the student makes the exchange for one of envelopes #3 or #4, what is the probability of the student then getting a hard exam?

66. At Awesome State University, grade inflation has become so extreme that atheletes are required to get A's in at least one half of their courses to remain elibible for team membership. A particular stellar athelete can choose to take four courses with probability 1/2 each to get an A or just three courses with probability 2/3 each for an A. Which choice offers the higher probability of meeting the eligibility requirement that an A be received in at least half of the courses taken?

67. Let X be a random variable on (0, 1) with uniform density function and let $Y = \tan(\pi (X - \frac{1}{2}))$.

   a) What is the probability that $Y > 1$.

   b) Give formulas for the density function and cumulative distribution functions for Y.

68. Two random variables, X and Y, both have uniform distributions on [0, 1]. This the case, the random variable Z is defined by $Z = \frac{X^2}{4Y+1}$.

   a) Indicate the region in the X-Y plane that corresponds to the event that $Z \leq \frac{1}{4}$. Then, compute the probability of this event.

   b) Set up and then evaluate an integral that gives the expectation of Z.

69. Suppose that A and B are events represented by disjoint subsets of a set where probabilities are assigned. Then, the probability of their union, $P(A \cup B)$, is equal to $P(A) + P(B)$. This understood, prove by induction that if you have n events, $A_1, \ldots, A_n$, represented by mutually disjoint sets, then $P(A_1 \cup \cdots \cup A_n) = P(A_1) + \cdots + P(A_n)$.

70. Indicate which of the following assertions are true and which are false. Give no explanation for your answers.
    a) If P is any probability measure, then $P(A \cup B) = P(A) + P(B)$ for any two events A and B.
    b) The number of distinct, '4 of a kind' poker hands is $13 \times 48$.
    c) Two standard dice are rolled and the sum is 6. The probability that one die or the other or both show 2 is $\frac{1}{5}$.
    d) A standard die is rolled seven times in succession. Of the number of sixes that can appear, 1 has the largest probability.
    e) The collection of all subsets of the positive integers is uncountable.
    f) Let p(w) denote the probability that the weight of a package of cheese at the supermarket is equal to w. The set of weights w where $p(w) > 0$ can be uncountable.
    g) Suppose that (X, Y) is a two dimensional random variable. Denote by F(s, t) the probability of the event that both $X < s$ and $Y < t$. Then, the probability that both $a < X < b$ and $c < Y < d$ is equal in all cases to $F(b, d) - F(a, c)$.

71. A chance device used by the Lottery Commission can generate any number between 2 and 30 for the "daily numbers game", with the probability of any individual number determined by a secret formula. However, the following are known to be true:

    Event A that the chosen number is prime has probability 0.4.
    Event B that the chosen number is less than 15 has probability 0.6
    Event C that the chosen number is a prime less than 15 has probability 0.3.

    a) Are events A and B independent? Explain?
    b) For each of the following events, specify the event in terms of A and B only. Then calculate its probability.
       1) The event that the number is a prime greater than or equal to 15."
       2) The event that the number is either prime or less than 15."
    c) If the chosen number is known to be prime, what is the probability that it is also less than 15?
    d) If the chosen number is known to be greater than or equal to 15, what is the probability that it is not prime?

72. Eleven cowhands have gathered in the Texahoma Saloon to play poker. They have only one deck of 52 cards, so they agree to deal 4-card rather than 5-card hands. How many distinct hands are there with
    a) all four cards of the same suit?
    b) three of a kind?
    c) two pairs?

In each case, you may leave your answer as a product of integers, but do not leave any binomial coefficients or factorials in the answer.

73. If a fair dice is rolled four times, what is the probability of a six appearing on precisely two of the rolls?

74. A certain digital watch model has a patented "random alarm" feature. After you push a button, the alarm goes off at a random time in the future. Consumers notice that this alarm often goes off an integral number of minutes after the alarm is pressed. One industry analyst publishes a hypothesis that the probability that the alarm goes off after precisely n minutes is equal to n/10. Explain why the analyst is wrong.

75. Suppose that X is a random variable with a uniform distribution in (0, 1]. Then, introduce the random variable $Y = \log(1/X)$ (natural logarithm).
    a) What is the probability that $2 < Y \leq 3$?
    b) Give the probability density function for Y.
    c) Calculate the expectation of Y.

76. Paul arrives in coffee hour after church on Epiphany Sunday and sees five pieces of "Three Kings cake," numbered 1 through 5, on a plate. He chooses piece #1 and is about to take his first bite when Elizabeth stops him. "There is something you should know about this cake," she says. "Two of the pieces, chosen at random, have metal stars baked into them (one small one S1 and one large one S2), and whoever gets a star is expected to throw a party for the people who baked the cake. You have to be careful when you bite into the cake because of the possibiility of hitting a star. I know where they are. Here, let me show you one." From the plate she selects the piece with the smallest available star, which happens to be piece #5, and shows the star to Paul.
    a) When the cake was baked, what is the probability that piece 1 had star S1 and piece 5 had star S2?
    b) When the cake was baked, what is the probability that the two stars were in pieces 1 and 5?
    c) Event B is "the smallest available star was in piece 5." Enumerate all the ways that this can occur (specifying in each case where the other star
    d) Event A is "Paul has a star in piece 1." Given that Event B has occurred, determine its probability?
    e) If Paul exchanges piece 1 for one of the other pieces of cake, what then is the probability that he has a star?
    f) What is the probability that the star that Elizabeth showed to Paul was star S1 rather than star S2?

77. Police in a suburb just west of Cambridge are rumored to stop cars along Concord Avenue at

random looking for minor safety code violations so as to generate income from the resulting traffic tickets. Moreover, the same rumor says that the police find that 10% of all cars stopped have such violations, and that 40% of the violators are under 20 years old while only 10% of the non-violators are under 20. Two drivers, Tuo and Fro, enter the suburb along Concord Avenue minutes of each other. Tuo is a teenager, but Fro is not. Assume the rumor is true.

a) What fraction of this suburb's Concord Avenue drivers are teenagers?
b) If the police stop Tuo, what is the probability that the officer will find a safety code violation?
c) If the police stop Fro, what is the probability that the officer will find a safety code violation?