ANSWERS:

1. a) $x - (2 - 2y + y^2)^{1/4} = 0$
   b) $(x^2 + 4)^{1/2} - 2y = 0$.
   c) $x^2/16 + y^2/9 = 1$.

2. a) $x = 1$.
   b) $27x + 4y = -73$.
   c) $x + 3y = -1$.

3. $e^2 + 1$.

4. a) $(t\cos(t^3), t\sin(t^3))$.
   b) $x\sin((x^2 + y^2)^{3/2}) - y\cos((x^2 + y^2)^{3/2}) = 0$.

5. a) **B** = 1/3 **A** + 1/3 (2, 4, -5).
   b) **B** = 11/25 **A** + 1/25 (92, 69, 25).
   c) **B** = -1/3 **A** + 1/3 (11, 8, 7).

6. Only in Case c) are **v** and **w** perpendicular.

7. a) 2/3.
   b) $(13)^{1/2}/7$.

8. a) $2x + y - z = 2$.
   b) $x + y - z = 3$.
   c) $x - y = -1$.

9. 6/7.

10. $t \to (t, t, t)$ or $t \to (-t, -t, -t)$.

11. a) Yes. b) No. c) Yes. The answer is yes if there is a constant, non-zero vector which is orthogonal to **v** at each time t. Otherwise, the answer is no. For a), consider (-1, 0, 5) and for c), consider (0, 7, 1). No such vector exists for b) since in this case, **v**(0) = (0, 0, 1) and so such a vector would have to lie in the x-y plane. But then it couldn't be simultaneously orthogonal to **v**(π/2) and **v**(-π/2).

12. a) In the first case, $L = 20x + y - z - 9$. In the second, $L = 20x - y + 3z - 7$.
    b) In the first case, $L = z$. In the second, $L = 3y + z$.

13. a) In the first case, the plane is where $z = 1$. In the second, it is where $x + z = 1$.
b) In the first case, the plane is where $2x - y = 2$. In the second, it is where $x + y - z = 1$.

14. $L = 2x - 5y + z$.

15. a) 2. b) 1. c) $-\sqrt{2}$.

16. $\nabla g = (-2, -4)$.

17. a) $\nabla f = (-\sin(x), 2y)$ so possible the critical points have the form $(n\pi, 0)$ with n an integer. The second derivative tests finds $(n\pi, 0)$ a local minimum when n is odd and a saddle when n is even.
b) $\nabla f = (-\sin(x)\sin(y), \cos(x)\cos(y))$ so the critical points have the form $(n\pi, (m + 1/2)\pi)$ and $((n + 1/2)\pi, m\pi)$ where n and m are integers. In the first case, if both n and m are odd or both are even, it is a local maximum. If one is odd and the other not, then it is a local minimum. In the second case, it is a saddle for all n and m. (Use the 2^{nd} derivative test.)
c) $\nabla f = (2x, 3y^2 - 3)$ so the critical points are $(0, \pm 1)$. The point $(0, 1)$ is a local minimum and the point $(0, -1)$ is a saddle.

18. The origin is the only interior critical point and it is a saddle. The extreme points on the boundary occur at $(\pm 3/\sqrt{2}, \pm\sqrt{2})$. The points $(3/\sqrt{2}, \sqrt{2})$ and $(-3/\sqrt{2}, -\sqrt{2})$ are maxima and the others are minima.

19. The maxima occur at $(\pm 1, 0, 0)$ and the minima at $(0, \pm 1, 0)$.

20. The maximum is at $(1, 0, 0)$.

21. The closest points are $(0, \pm 1, \pm 1)$ with distance $\sqrt{2}$.

22. The square of the length of **E** is $x^4 + 4y^2 + z^2 + z^2y^2 - 2zy + 1$ which is smallest at the origin.

23. $3(\pi/2 - 1)$.

24. $4/(3\pi)$. (Remember to divide the integral of x by the area.)

25. $\pi/8$.

26. $\sin(1)/2$.

27. $\pi/2$

28. $\pi \sin(1)$.

29. $4\pi\,(1 - \cos(1))/3$.

30. $\int_0^1 \left(\int_0^r \left(\int_0^\pi \left(r^3 \sin(\theta)\cos^2(\theta)\right) d\theta \right) dz \right) r\,dr = 1/9.$

31. $\int_0^{p/2} \int_0^{p/4} \left(\int_0^{1/\cos(\varphi)} \left(\rho^4 \sin(\theta)\cos(\theta)\sin^2(\phi)\cos^2(\phi)\right)\rho^2 d\rho \right) \sin(\phi)\,d\phi\,d\theta = 1/56.$

32. $\int_{-3/5}^{3/5} \int_{-\sqrt{9/25-x^2}}^{\sqrt{9/25-x^2}} \int_{\frac{4}{3}\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz\,dy\,dx =$

$\int_0^{4/5} \left(\int_{\frac{3}{4}r}^{\sqrt{1-r^2}} \left(\int_0^{2\pi} d\theta \right) dz \right) r\,dr =$

$\int_0^1 \left(\int_0^{\mathrm{Arc\,tan}(4/3)} \left(\int_0^{2\pi} d\theta \right) \sin(\phi)\,d\phi \right) \rho^2\,d\rho = 4\pi/15.$

33. a) $\int_0^\infty \int_0^{2\pi} e^{-r^2}\,d\theta\,r\,dr = \pi.$

b) The square of this integral is the Cartesian coordinate version of the preceding integral, so the integral in question equals $\pi^{1/2}$.

34. $u(t, x) = e^t\,(1 + (x - 4t)^2)^{-1}$.

35. a) Advection. b) Advection. c) Diffusion.

36. 0

37. $-2/\pi - 2/7$

38. 0.

39. a) $\mathbf{X}(u, v) = (2\,(1 - u^2/9 - v^2/25)^{1/2}, u, v)$ for values of (u, v) with $u^2/9 + v^2/25 \le 1$.

b) $\mathbf{X}(u, v) = (u, 3\,(1 - u^2/4 - v^2/25)^{1/2}, v)$ for values of (u, v) with $u^2/4 + v^2/25 \le 1$.

c) $\mathbf{X}(u, v) = (1 - u^2 - v^4, u, v)$ for values of (u, v) with $u^2 + v^4 \le 1$.

d) $\mathbf{X}(u, v) = (u, v, 2 - u - v)$ for values of (u, v) in $u \ge 0$, $v \ge 0$ and $u + v \le 2$.

40. a) $\int_{-5}^{5}\left(\int_{-3\sqrt{1-v^2/25}}^{3\sqrt{1-v^2/25}} \sqrt{1+4(u^2/81+v^2/625)/(1-u^2/9-v^2/25)}\,du\right)dv.$

b) $\int_{-5}^{5}\left(\int_{-2\sqrt{1-v^2/25}}^{2\sqrt{1-v^2/25}} \sqrt{1+9(u^2/16+v^2/625)/(1-u^2/4-v^2/25)}\,du\right)dv.$

c) $\int_{-1}^{1}\left(\int_{-\sqrt{1-v^4}}^{\sqrt{1-v^4}} \sqrt{1+4u^2+16v^6}\,du\right)dv.$

41. 10/3. (The area of the surface is π and the integral of z over the surface is 10π/3.)

42. sin(1).

43. 500 π/3.

44. (x, -y, 0).

45. a) (x^2yz/2, 0, 0).
b) (2z, 3x, y).

46. a) (0, x/π).
b) (x, 0, 0).

47. a) No such vector field exists because (x, -2y, xy) has divergence -1 and the divergence of a curl is zero.
b) (xy cos(yz^2), 0, 0).

48. 5x + 3y = 0.

49. $\iint_S$ x g dS. Let **E** = (g, 0, 0); here div(**E**) = f while **E•n** = x g on the surface of the volume, V, in question. Thus, the divergence theorem implies that $\iint_S$ x g dS = $\iiint_V$ f dV.

50. a) 2π . This is a direct computation: Parameterize the circle by t → (cos(t), sin(t)) and then path integral is just the integral of dt between 0 and 2π.
b) 2π. Use Green's theorem for a region with holes and note that the vector field in question has zero 'curl' in the sense that when written as (P, Q), then Q_x - P_y = 0.
c) 0. This is also a direct application of Green's theorem.

51. A = (-1, 0, 0, 0, 3), B = (-1, -1, 0, 2, 2).

52. The mean is $m = (n_1 m_1 + n_2 m_2)/(n_1 + n_2)$ and the standard deviation is
$s = ((n_1 - 1) s_1^2 + (n_2 - 1) s_2^2)^{1/2}/(n_1 + n_2 - 1)^{1/2}$.

53. For 3.18: This is $Pr(B \cup C) = Pr(B) + Pr(C) – Pr(B \cap C) = .099$

For 3.19: This is $Pr(A \cup B \cup C) = .0143$.

For 3.20: This is $Pr(A \cap \overline{B} \cap \overline{C}) + Pr(\overline{A} \cap B \cap \overline{C}) + Pr(\overline{A} \cap \overline{B} \cap C) = .1368$

For 3.21: This is Pr(affected person is female) = .677.

For 3.22: This is Pr(both affecteds are female) = .263.

For 3.23: This is Pr(both < 80) = .160.

54. For 3.85: Use Baye's theorem to find $Pr(Y_1|(X_1 \cap X_6 \cap X_4)) = .009$.

For 3.87: You are computing $Pr(X_7|Y_2) = .7$.

For 3.88: Youi are computing $Pr(\overline{X}_7|\overline{Y}_2) = .605$.

55. For 3.104: .938.

For 3.105: .988.

56. For 4.37: Use the binomial distribution to find that the answer is .172.

57. For 4.44: The probability of the 82 year old dying in the next year is $p_1 = (l_{82} – l_{83})/l_{82} = .104$. Similar probabilities, $\{p_j\}_{2\le j\le 11}$ can be obtained for the others. The sum, $\sum_{1\le j\le 11} p_j = .176$, is the answer.

58. For 4.69: Use the binomial expansion with n = 5, p = .4 to find Pr(X = 3) = .230.

For 4.70: $Pr(X \ge 3) = .317$ using binomial table (Table 1) in Chapter 4.

59. For 4.78: Use the Poisson distribution with $\mu = 15.6$ to find the answer $\cong 7.651 \times 10^{-13}$.

60. For 5.31: This is given by $\Phi(-1.667) = 1 - \Phi(1.667) \cong .048$.

For 5.32: This is given by $\Phi(-3) = 1 - \Phi(3) \cong .0013$.

61. For 5.61: This is $84!/(29! \times 55!) (.24)^{29} (.76)^{55} \cong .009$

For 5.62: Use the fact that $Pr(X \ge 29) \cong Pr(Y \ge 28.5)$ where Y is normally distributed with mean = np = 20.16 and variance = npq = 15.32. Thus, $Pr(Y \ge 28.5) \cong .017$.

62. For 5.64: $Pr(X \ge 90) = 1 - \Phi(2.307) = .0105$.

For 5.65: Approximate the binomial distribution with a normal one of mean np = 21.1 and variance npq = 20.8. Then, $Pr(X \ge 25) \cong 1 - \Phi(.0755) = .225$.

63. a1) $A' \cap C$ and it has size 3750.

a2) $A \cap B' \cap C'$ and it has size 650.

b) $A \cap D \subseteq C' \cap B$.

c) Probability = $\frac{600+3,750}{10,000}$ = 0.435

64. a) $\binom{13}{5} = 13 \times 11 \times 9$.

b) $\binom{13}{5} \times \binom{13}{3} \times \binom{13}{3} \times \binom{13}{2} = 13 \times 11 \times 9 \times (13 \times 11 \times 2)^2 \times 13 \times 6$.

c) $12 \times$ (Answer to Part b).

65. a) To parse the logic, imagine that the Course Head labeled the easy exams E1 and E2. The possibilities for event B are:

H1 H2 E1 E2
H1 H2 E2 E1
H2 H1 E1 E2
H2 H1 E2 E1
E1 H1 E2 H2
E2 H1 E1 H2
E1 H1 H2 E2
E2 H1 H2 E1

Each of these cases has probability $\frac{1}{24}$ since there are 24 possible ways to distribute the exams. WIthot distinguishing the easy exams, then the cases are

H1 H2 E E
H2 H1 E E
E H1 E H2
E H1 H2 E

Each of these has probability $\frac{1}{12}$ since each corresponds to two distinct cases in the first list above.

b) The conditional probability of A occurring given B is that of obtaining either of the first two distributions in the second list above out of the four equally probable distributions. Thus, this probability is $\frac{1}{2}$.

c) This is the conditional probability of A occurring given B, so equals $\frac{1}{2}$.

d) Suppose that the student selects envelope 3. One is asked to compute here the conditional probability for envelope 3 to have a hard exam given that event B occurs. As indicated by the second list, there is one way for this to happen (the final distribution) out of four equally likely possibilities for event B. Thus, the probability is $\frac{1}{4}$.

66. Taking 4 courses, the probability of 2 A's is $6 \times (\frac{1}{2})^2 \times (\frac{1}{2})^2 + 4 \times (\frac{1}{2})^3 \times \frac{1}{2} + (\frac{1}{2})^4 = \frac{11}{16}$.
Taking 3 courses, the probability is $3 \times (\frac{2}{3})^2 \times \frac{1}{3} + (\frac{2}{3})^3 = \frac{20}{27}$, so 3 courses is better.

67. a) The probability that $Y > 1$ is the probability that $\pi (X - \frac{1}{2}) > \frac{\pi}{4}$ which is that of $X > \frac{3}{4}$. This is $\frac{1}{4}$.

b) The probability that $Y < t$ is that of $X < \frac{1}{2} + \frac{1}{\pi}\arctan(t)$, so $F_Y(t) = \frac{1}{2} + \frac{1}{\pi}\arctan(t)$. The corresponding density function is the derivative of $F_Y(t)$, namely $\frac{1}{\pi(1+t^2)}$.

68. a) The region corresponds to the portion of the unit square in the X-Y plane that lies above the parabola $Y = X^2 - \frac{1}{4}$. The corresponding probability is the area of this region, $\frac{5}{6}$.

b) The expectation of Z is the integral $\int_0^1\int_0^1 \frac{X^2}{4Y+1}\,dY\,dX = \frac{1}{3} + \frac{1}{4}\ln(5)$.

69. The result is true by assumption for $n = 2$. Now, assume that the result is true for all integers less than some integer $n > 2$. Then, write $P(A_1 \cup \cdots \cup A_n) = P((A_1 \cup \cdots \cup A_{n-1}) \cup A_n)$ and invoke the $n = 2$ case using $A = A_1 \cup \cdots \cup A_{n-1}$ and $B = A_n$. This then proves the equality $P(A_1 \cup \cdots \cup A_n) = P(A_1 \cup \cdots \cup A_{n-1}) + P(A_n)$. Now invoke the induction hypothesis to write $P(A_1 \cup \cdots \cup A_{n-1})$ as the sum of the probabilities of the A_k's for $k \le n-1$ to complete the proof for the integer n case.

70. a) False. (Take the case A = B.)

b) False. (The number is 13.)

c) False. (The probability is $\frac{11}{36}$.)

d) True. (As the expected number is 7/6, the only issue is whether 0 sixes has larger probability than 1. This isn't the case as the former has probability $(\frac{5}{6})^7$ and the latter $\frac{7}{6}(\frac{5}{6})^6$.)

e) True. (The collection of all <u>finite</u> subsets is countable.)

f) False. (For each positive integer n, let A_n denote the set of weights w where $2^{-n} < p(w) \le 2^{-(n-1)}$. Then, the number of weights in A_n is at most 2^n. As the set of weights with $p(w) > 0$ is $\cup_n A_n$, a countable union of countable sets, this set is countable.)

g) False. (The probability is $F(b, d) - F(a, c) - F(b, d) + F(a, c)$.)

71. a) The events are not independent since $P(A \cap B) = 0.3$ while $P(A) \times P(B) = \frac{6}{25}$.

b1) $A \cap B'$. The probability is $P(A) - P(C) = 0.1$.

b2) $A \cup B$. The probability is $P(A) + P(B) - P(C) = 0.7$.

c) $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{3}{4}$.

d) $P(A'|B') = \frac{P(A'\cap B')}{P(B')} = \frac{1-P(A\cup B)}{1-P(B)} = \frac{3}{4}$.

72. a) $4 \times \binom{13}{4} = 13 \times 11 \times 10 \times 2$.

b) 13×4.

c) $13 \times 12 \times 6 \times 3$.

73. $\binom{4}{2} \times (\frac{1}{6})^2 \times (\frac{5}{6})^2 = \frac{25}{216}$.

74. The sum of the probabilities is greater than 1.

75. a) $e^{-2} - e^{-3}$.
    b) The probability that Y is less than t is that of $X > e^{-t}$, so Y has distribution function $1 - e^{-t}$. Thus, the density function for Y is e^{-t}.
    c) The expectation of Y is the integral of $t\,e^{-t}$ from 0 to ∞. This is 1.

76. a) $\frac{1}{20}$
    b) $\frac{1}{10}$
    c) The possible cake numbers for S1 and S2 are (1, 5), (5, 1), (5, 2), (5, 3), (5, 4).
    d) $\frac{2}{5}$.
    e) $\frac{1}{5}$.
    f) $\frac{4}{5}$.

77. a) 13%.
    b) $\frac{4}{13}$
    c) $\frac{2}{29}$