Some True-False Review Problems

Mark with a true assertion below with a T and a false one with F.

1. If u(x, y, z) obeys $u_x + u_y + u_z = 3$, then u cannot have a saddle point.

2. The vector **v** = (2, -2, 1) is normal to the level surface of $f(x, y, z) = 2x^2 + y^2 + z^2 – 7$ at (1, 2, 1).

3. Some level surfaces of $f(x, y, z) = \sin((4x + 2y + z)^2)$ are spheres.

4. The minimum value of $f(x, y) = (x – 2)^2 + (y – 3)^2$ where $x^2 + y^2 \leq 1$ is achieved on the boundary where $x^2 + y^2 = 1$.

5. The best linear approximation to $f(x, y) = 2x^2 + y^3$ near (1, 1) is L(x, y) = 4x + 3y – 4.

6. If h(x, y) = f(x) g(y) and R is the region where both $0 \leq x \leq 1$ and $0 \leq y \leq 1$, then the integral over R of h is the product of the integrals of f(x) and g(y), both from 0 to 1.

7. If $f(x, y) = e^{\cos(2x-y)} (2x – y)^4$ and $\mathbf{u} = \frac{1}{\sqrt{5}}(2, 1)$, then the directional derivative of f in the direction of **u** is zero at all points.

8. For any function f(x, y), the equality $\int_0^1 \left[\int_0^{2-2x} f(x,y)dy \right] dx = \int_0^2 \left[\int_0^{1-y/2} f(x,y)dx \right] dy$ holds.

9. The function $(x^2 + y^2)^{1/2}$ has a global minimum at x = y = 0.

10. If $f(x, y) = g(x^2 – y^2)$ where g is any function of one variable, then $y f_x = x f_y$.

11. If gx, y) is any function of two variables and if R is the square where both $0 \leq x \leq 1$ and $0 \leq y \leq 1$, then $\int_R g_{xy}\, dA = g(1, 1) + g(0, 0) – g(1, 0) – g(0, 1)$.

12. The only solution to the equation $u_t = 2 u_x$ is u(t, x) = 2t + x.

13. If f and g are any two functions of one variable and u(t, x) = f(x + 3t) + g(x – 3t), then u satisfies $u_{tt} = 9 u_{xx}$.

Answers

1. True. In fact, such a function does not have any stationary points.

2. False. The vectors proportional to (4, 4, 2) are normal at that point.

3. False. All level surfaces are planes of the form $4x + 2y + z = \text{constant}$.

4. True. The minimum is at $\frac{1}{\sqrt{13}}(2, 3)$.

5. True.

6. True.

7. False. The gradient of f is proportional to (2, -1) whose dot product with **u** is not zero.

8. True.

9. True.

10. False. In fact, $y f_x = - x f_y$.

11. True. For a proof, do the x-integral first and integrate by parts. Then, do the y-integral and integrate by parts.

12. False. Any u(t, x) of the form h(2t + x) satisfies this equation.

13. True.