Answers to First Math 21a Practice Hourly 1

1. a) The line traversed can be parametrized by $t \to (-2 + 3t, 8 - 2t, -6 + 2t)$.
   b) The surface is reached when the z-coordinate, $-6 + 2t$ is zero. Thus, $t = 3$ and the coordinates of the point in question are (7, 2, 0).
   c) The closest point has distance 6 from the origin. (The point is (4, 4, -2).)

2. a) The bug's velocity is $\mathbf{r}' = 2\,(\frac{1}{t}, t, -\sqrt{2})$.
   b) The bug's speed is $|\mathbf{r}'| = 2\,(\frac{1}{t} + t) = \sqrt{4(\frac{1}{t^2} + t^2 + 2)}$.
   c) The path length is the integral from 1 to 2 of $|\mathbf{r}'|$ which is $3 + 2\ln(2)$.
   d) The component is $\frac{1}{3}(1 - 2\sqrt{2})\,(1, 1, 1)$.

3. a) False, as $\mathbf{w} = 0$ when either $\mathbf{u}$ or $\mathbf{v}$ is zero, or when $\mathbf{v} = r\,\mathbf{u}$ with $r > 0$.
   b) True, as the dot product between these two vectors is zero.
   c) False, as $\mathbf{w}$ is parallel to the x-axis when $\mathbf{u} = (1, 0, 0)$ and $\mathbf{v} = (-1, 0, 0)$.

4. a) The line intersects the plane at the point $\frac{1}{21}(267, -40, 2)$.
   b) The distance from the origin to Π is 9.
   c) $-\frac{28}{3}$.

5. In Cartesian coordinates, $r = 2\cos\theta$ reads $\sqrt{x^2 + y^2} = 2x/\sqrt{x^2 + y^2}$. Multiply through by $\sqrt{x^2 + y^2}$ to find that $x^2 + y^2 - 2x = 0$. Add 1 to each side to find that $x^2 - 2x + 1 + y^2 = 1$ which is to say, $(x - 1)^2 + y^2 = 1$. Thus, the circle has its center at the point (1, 0) and its radius is 1.

6. a) If $\mathbf{u}$ is tangent to Π, then $\mathbf{u}$ is perpendicular to a normal vector to Π. In our case, the vector $\mathbf{n} = (1, 1, 1)$ is a normal vector to Π since it is perpendicular to both the vector that points from from A to B (which is (-1, 1, 0)) and the vector, (-1, 0, 1), which points from A to C. Meanwhile, $\mathbf{u}\cdot\mathbf{n} = 1 + 2 - 3 = 0$.
   b) $\mathbf{v} = \mathbf{n} \times \mathbf{u} = (-5, 4, 1)$ is such a vector.
   c) The vector $\mathbf{w} - |\mathbf{u}|^{-2}(\mathbf{w}\cdot\mathbf{u})\,\mathbf{u}$ is orthogonal to $\mathbf{n}$ since both $\mathbf{w}$ and $\mathbf{u}$ are. Also, it is orthogonal to $\mathbf{u}$ since its dot product with $\mathbf{u}$ is zero. Thus, it must be a multiple of $\mathbf{v}$ since $\mathbf{v}$ is also orthogonal to both $\mathbf{u}$ and $\mathbf{n}$. You can think of $\mathbf{n}$ and $\mathbf{u}$ as pointing along two orthogonal axis in space and then $\mathbf{v}$, being orthogonal to both $\mathbf{n}$ and $\mathbf{u}$, lies along the third axis. As $\mathbf{w}$ has zero dot product with both $\mathbf{n}$ and $\mathbf{u}$, like $\mathbf{v}$, it is parallel to the third axis and so is a multiple of $\mathbf{v}$.