Math 21a second practice exam for Hourly 2

(Fall 2001)

Solutions

1. a) $\nabla f = (3, 4)$.
   b) The tangent plane is given by $z - 3x - 4y = -5$.
   c) Using the linear approximation, $f(1.02, 2.05) = f(1, 2) + \nabla f|_{(1,2)} \cdot (.02, .05) = 6.26$.

2. The Lagrange equations are $-\log(x) - 1 - a = \lambda, -\log(x) - 1 - b = \lambda, -\log(z) - 1 - c = \lambda, x + y + z = 1$. We have $x = e^{-(\lambda+1)}e^{-a}, y = e^{-(\lambda+1)}e^{-b}, z = e^{-(\lambda+1)}e^{-c}, e^{-\lambda+1)}(e^{-a} + e^{-b} + e^{-c}) = 1$. The stationary points occur at $(e^{-a}, e^{-b}, e^{-c})/(e^{-a} + e^{-b} + e^{-c})$.

3. $8/3$.

4. a) The stationary points are $(0, 1), (0, -1), (1, 1), (1, -1), (-1, 1), (-1, -1)$.
   b) The local maximum is $(0, -1)$. The local minima are $(1, 1), (-1, 1)$. The remaining three are saddles.
   c) If the level set is tangent to the y axis, then ∇f is orthogonal to $(0, 1)$ and so $f_y = 0$. This occurs where $y = \pm 1$. Where $y = 1$, $f = x^4 - 2x^2 - 6$ and so $f = 2$ if $x = \pm 2$. Where $y = -1, f = x^4 - 2x^2 - 6$ and so f is not equal to 2 for any value of x. Thus, the points are $(2, 1)$ and $(-2, 1)$.

5. Change the order of integration to write this integral as $\int_0^{\pi/2} \left(\int_{\sin(y)}^1 dx \right) dy = \pi/2 - 1$.

6. Let z denote height, x denote length and y denote width. The you are asked to minimize the function $f(x, y, z) = 50xy + 20(xz + yz)$, where x, y and z are constrained by the requirement $xyz = 20,000,000$. The minimum has $z = 500$ centimeters and $x = y = 200$ centimeters.