

Math 21a answers for Hourly 2 practice problems

1. a) $L(x, y, z) = 2x + 2y + 2z - 3$. b) $L(x, y, z) = y + z$.
2. 12 (Note: Since $y_r = 0$ at $r = 1$ and $s = -1$, you don't need to compute any y derivatives when using the Chain Rule.)
3. $\pi + 1$. ($w' = 4t \tan^{-1}(t) + 1$ at $t = 1$).
4. $\nabla f = (3, 2, -4)$.
5. 2
6. $\mathbf{u} = \frac{1}{\sqrt{3}}(1, 1, 1)$.
7. 0.
8. $2x + 2y + z - 4 = 0$.
9. $x - y + 2z - 1 = 0$.
10. $2\sqrt{2}$.
11. Absolute max: 1 at $(0, 0)$; absolute min: -5 at $(1, 2)$. Note that this is a tricky question as you have to check the corner points by hand.
12. Absolute max: 4 at $(2, 0)$; absolute min: $3/\sqrt{2}$ at $(3, \pi/4)$, $(3, -\pi/4)$, $(1, \pi/4)$ and $(1, -\pi/4)$.
13. Local min at $(1, -2)$; saddle point at $(-1, -2)$.
14. Length in the x -direction: $4\sqrt{2}$; length in the y -direction: $3\sqrt{2}$.
15. $(3/2, 2, 5/2)$.
16. $(0, 0, 2)$ and $(0, 0, -2)$.
17. $(\pm 4/3, -4/3, -4/3)$.

$$18. \begin{pmatrix} -30x + \frac{2-2x^2}{(1+x^2)^2} & 1 \\ 1 & 0 \end{pmatrix}$$

$$19. L(x, y, z) = x + y - z - 1.$$

$$20. w = (b^2 V/(ac))^{1/3}; d = (c^2 V/(ab))^{1/3}; h = (a^2 V/(bc))^{1/3}.$$

21. A normal vector to the surface is the gradient of $xz^2 - yz + \cos(xy) - 1$ which, at $(0, 0, 1)$, is the vector $\mathbf{v} = (1, -1, 0)$. Meanwhile, $\mathbf{r}(1) = (0, 0, 1)$ and $\mathbf{r}'(1) = (1, 1, 1)$ which has zero dot product with the normal vector $\mathbf{v}$. Thus, $\mathbf{r}'(1)$ lies in the tangent plane to the surface.

22. Set $z(x, y) = x^3 + y^3 - 9xy + 27$ as a function on $\mathbb{R}^2$. Then, maxima and minima can only occur at points where ∇z is zero. Since $\nabla z = (3x^2 - 9y, 3y^2 - 9x, 0)$, this occurs where $x^2 = 3y$ and $y^2 = 3x$. Thus, at $(x, y) = (0, 0)$ and $(x, y) = (3, 3)$. The second derivative test for z'' at the point $(0, 0)$ finds z'' to have only the off diagonal entries non-zero. (These equal 9). Thus, $\det(z''(0, 0)) < 0$ so $(0, 0)$ is a saddle. At $(3, 3)$, $\det(z'') = 243 > 0$ and $\text{trace}(z'') = 36$ so $(3, 3)$ is a local minimum.

$$23. 1.$$

$$24. (3 \ln 2)/2.$$

$$25. 2.$$

$$26. (e - 2)/2.$$

$$27. 12.$$

$$28. -48.$$

$$29. 16.$$

$$30. 1/75$$