Math 21a Study Guide for Hourly 2
What follows are a list of topics and issues that may arise on the second hourly exam. Though not exclusive, the list is provided to help you identify some of the salient topics that the course covers in Chapters 2, 3.1-3.2, 4.4 of the text book and the supplement on Lagrange Multipliers. Please note that the list below is not exclusiveyou may be asked about material that is not listed here.

In any event, remember that the second hourly exam is on Wednesday, November 14, from 7:30-9:30pm in Science Center lecture halls C and E (go to either one). Please arrive a few minutes before 7:30 as we will start timing the exam at 7:30.

- Be able to compute the gradient, matrix of second derivatives and higher order derivatives of a given function of two or three variables.

- For a function of three variables, construct the best approximating function near a specified point (x_0, y_0, z_0) that is linear in $(x - x_0)$, $(y - y_0)$ and $(z - z_0)$.Be able to do make the analogous construction for a function of two variables.

- Give a formula for the tangent plane to a particular level surface of a specified function f(x,y,z) at a given point. Find a formula for the normal vector to the level surface at that point. For a function of two variables, be able to give an equation for the tangent line to a given level curve at a given point.

- Understand the relationship between the level surface of a given function f(x,y,z) and the level surface of the best linear approximation to this function at a given (x_0, y_0, z_0).

- Be able to compute directional derivatives of functions of two and three variables and to use these derivatives to find the rate of change of a function along a given parameterized curve.

- Given a function of two or three variables, locate all of its stationary points and determine which (if any) are global maxima or global minima of the function.

- Know how to use the matrix of second derivatives of a function of two variables to decide whether a stationary point is a saddle, local maximum or local minimum. Identify the cases where this matrix cannot distinguish saddles, minima or maxima.

- Sketch or identify level curves of a function of two variables in the vicinity of a stationary point.

- Use the method of Lagrange multipliers to find the stationary points of a function, f, of two or three variables subject to a constraint given by requiring that points move only along a given level curve or surface of another function, g.

- For example, be able to use the Lagrange multiplier technique to find the closest and farthest points on a level surface g(x,y,z) = c from a given point.

- Understand the geometric meaning of the Lagrange multiplier condition. For example, given a specific level curve of a function g(x,y), and a sketch that includes that level curve and also level curves of some other given function f(x, y), be able to identify on the sketch the points along the level curve of g where f might have constrained local maxima or minima.

- Find all global maxima and minima of a specified function of 2 or 3 variables in a given region with boundary. Thus, be able to identify the stationary points both in the interior of the region and on the boundary.

- Given a function f of u,v and maybe w, and given that u, v and w are specified functions of other variables, x, and maybe y and z, be able to use the chain rule to verify asserted relationships between various partial derivatives of the new function h(x, y, z) = f(u(x,y,z), v(x,y,z), w(x, y, z)).

- For example, know how to use the chain rule to determine the numerical values of the partial derivatives of the function $h(x, y, z) = f(u(x, y, z), v(x, y, z), w(x, y, z))$ at a given point in the situation when the numerical values of the partial derivatives of f, u, v and w are known at appropriate points, but not their functional form.

- Write down or identify a Riemann sum formula that approximates a double or triple integral of a specified function over a specified region in either the plane or space.

- Understand how the integral of a specified function of two or three variables over a specified region relates to the average value of that function.

- Given a specific iterated integral for a function of two variables, be able to identify the domain of integration (i.e., the region of the plane over which the integral takes place.)

- Be able to write down an iterated integral that computes the integral of a given function over a given region of the plane. For example, if the region consists of those points where $a \leq x \leq b$ and $g^-(x) \leq y \leq g^+(x)$ be able to write down the corresponding iterated integral. Likewise, if the region is given as $a \leq y \leq b$ and $h^-(y) \leq x \leq h^+(y)$.

- If the integral of a function over a given region of the plane is expressed as an iterated integral with the y-integration first, be able to re-express the integral as one with the x-integration done first. The problem here is that of determining the limits of integration for the x and y variables in the case where the x integral is done first from those given in the case where the y integral is done first.

- Be able to write down an iterated integral that computes the integral of a given function of three variables over a given region in space.