Suggested problems for the Differential Equation lectures

Suggested problems for the 12/3 lecture

1. Find the general form for the solution, p(t), to the following equations:
   a) $p' = c\, p^2$ where c is a constant.
   b) $p' = c\, p^m$ where c is a constant and m is an integer.

2. The equation $p' = p\,(1 - p)$ is known as a 'logistics equation'. Its general solution can be found by the method that is described in Section 1c of the Differential Equation supplement.
   a) Verify by plugging into the logistic equation that $p(t) = a\, e^t/(1 + a\, e^t)$ is a solution for any constant a.
   b) Prove that there is a solution of the logistics equation whose time t = 0 value can be any given real number. That is, given any real number p_0, find a solution p(t) to the logistics equation with $p(0) = p_0$.
   c) Write down the version of Equation (2.18) of the Differential Equation supplement when $f(x) = x\,(1 - x)$ comes from the logistics equation and where $x_0 = 0$. Then, write down the solution, x(t), to the resulting equation that satisfies x(0) = .001.
   d) Use p(t) to denote the logistics equation solution that has p(0) = .001; and let x(t) be your solution from the Part c of this problem. Give a rough estimate for $|p(t) - x(t)|$ when $t < 1$.

Suggested problems for the 12/5 lecture

1. Find the solutions to the equation

$$\frac{\partial}{\partial t} u(t, x) = -2 \frac{\partial}{\partial x} u(t, x) + u(t, x)$$

   which obey the following:
   a) u(0, x) = cos(x).
   b) u(0, x) = sin(x).
   c) u(1, x) = cos(x).
   d) u(2, x) = cos(x).

2. Verify by taking the appropriate derivatives that the function in Equation (4.19) of the

Differential Equation supplement obeys Equation (4.7) in the same supplement.

3. First, give a heuristic argument for the claim that <u>any</u> equation that is purported to model the time and or space evolution of the population of <u>non</u>-<u>interacting</u> entities should obey the superposition principle: *If* u_1 *and* u_2 *are solutions to the equation in question and if* a_1 *and* a_2 *are constants, then* $u = a_1 u_1 + a_2 u_2$ *is also a solution to the given equation.* Next, having made your argument, reread Section 1d of the Differential Equation supplement and consider the exponential growth equation $p' = a\,p$ with a = constant. Do you expect this equation to obey the superposition principle? Does it?

Suggested problems for the 12/7 lecture

1. Use (5.7) in the Differential Equation supplement to find a solution, u(x), to the equation $-u_{xx} = k$ in the case where k(x) is given by

$$k(x) = \begin{Bmatrix} 0 & \text{when } x < 0 \\ e^{-x} & \text{when } x \geq 0 \end{Bmatrix} .$$

2. Fix a vector $\vec{s}$ in $\mathbb{R}^3$ and then consider $G = \frac{1}{4\pi}\frac{1}{|\vec{r} - \vec{s}|}$ as a function of $\vec{r} = (x, y, z)$.

Take the appropriate derivatives to verify that $G_{xx} + G_{yy} + G_{zz} = 0$ where $\vec{r} \neq \vec{s}$.