

SOLUTIONS TO PROBLEMS IN CURVES, TANGENTS AND DIFFERENTIAL EQUATIONS	Math21a, Fall 2002
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SHORT SOLUTIONS TO PROBLEMS (should already be enough for full credit)

Problem 1. b is a parameter telling how much the predator eats.

Problem 2. By differentiaton, we check that $x''(t) + 2x'(t) - 2x(t) = 0, y''(t) + 2y'[t] - 2y(t) = 0$.

Problem 3. Except for the initial condition $x = \pi$, the solutions will spiral towards the position $x = 0$ which is the same as $x = 2\pi$.

MORE DETAILED SOLUTIONS TO PROBLEMS.

Problem 1)

The parameter b in the population model

$$\begin{aligned} \dot{x} &= 0.4x - bxy \\ \dot{y} &= -0.1y + 0.2xy \end{aligned}$$

represents the effect of the predation the pray. In other words, it depends on how much each predaors eats. If b is larger, then this will reduce the amount of pray and consequently also the amount of predators. The stable point, where predators and prays don't change in time will be lower and more to the left.

Problem 2)

Solution. To obtain the velocity and acceleration of the curve, we differentiate $x(t) = e^{-t} \cos(t), y(t) = e^{-t} \sin(t)$ once and twice, we get $x'(t) = e^{-t}(-\sin(t) - \cos(t)), y'(t) = e^{-t}(\cos(t) - \sin(t)), x''(t) = 2e^{-t} \sin(t), y''(t) = 2e^{-t} \cos(t)$.

We see that $x''(t) + 2x'(t) - 2x(t) = 0, y''(t) + 2y'[t] - 2y(t) = 0$.

Problem 3)

The pendulum will in general settle down to the bottom position, where the angle is 0. The speed will decrease also. The circular curves in the frictionless case will be replaced by spiral type curves moving towards the position $(0, 0)$ rsp. $(0, 2\pi)$. In the exceptional case, where the pendulum starts at the very top $(x = \pi)$, it will stay in that unstable position.

