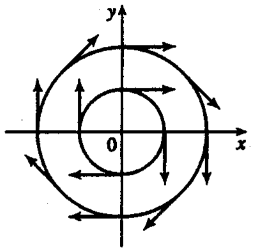
HW 10b

6. $\mathbf{F}(x,y) = \dfrac{y\,\mathbf{i} - x\,\mathbf{j}}{\sqrt{x^2+y^2}}$

All the vectors $\mathbf{F}(x,y)$ are unit vectors tangent to circles centered at the origin with radius $\sqrt{x^2+y^2}$.



29. $f(x,y) = xy \;\Rightarrow\; \nabla f(x,y) = y\,\mathbf{i} + x\,\mathbf{j}$. In the first quadrant, both components of each vector are positive, while in the third quadrant both components are negative. However, in the second quadrant each vector's x-component is positive while its y-component is negative (and vice versa in the fourth quadrant). Thus, ∇f is graph IV.

30. $f(x,y) = x^2 - y^2 \;\Rightarrow\; \nabla f(x,y) = 2x\,\mathbf{i} - 2y\,\mathbf{j}$. In the first quadrant, the x-component of each vector is positive while the y-component is negative. The other three quadrants are similar, where the x-component of each vector has the same sign as the x-value of its initial point, and the y-component has sign opposite that of the y-value of the initial point. Thus, ∇f is graph III.

31. $f(x,y) = x^2 + y^2 \;\Rightarrow\; \nabla f(x,y) = 2x\,\mathbf{i} + 2y\,\mathbf{j}$. Thus, each vector $\nabla f(x,y)$ has the same direction and twice the length of the position vector of the point (x,y), so the vectors all point directly away from the origin and their lengths increase as we move away from the origin. Hence, ∇f is graph II.

32. $f(x,y) = \sqrt{x^2+y^2} \;\Rightarrow\; \nabla f(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}\,\mathbf{i} + \dfrac{y}{\sqrt{x^2+y^2}}\,\mathbf{j}$. Then

$|\nabla f(x,y)| = \dfrac{1}{\sqrt{x^2+y^2}}\sqrt{x^2+y^2} = 1$, so all vectors are unit vectors. In addition, each vector $\nabla f(x,y)$ has the same direction as the position vector of the point (x,y), so the vectors all point directly away from the origin. Hence, ∇f is graph I.

10. $\int_C yz\,dy + xy\,dz = \int_0^1 (t)(t^2)\,dt + \int_0^1 \sqrt{t}\,(t)\,2t\,dt = \int_0^1 \left(t^3 + 2t^{5/2}\right) dt = \left[\frac{1}{4}t^4 + \frac{4}{7}t^{7/2}\right]_0^1 = \frac{23}{28}$

14. Vectors starting on C_1 point in roughly the same direction as C_1, so the tangential component $\mathbf{F}\cdot\mathbf{T}$ is positive. Then $\int_{C_1} \mathbf{F}\cdot d\mathbf{r} = \int_{C_1} \mathbf{F}\cdot\mathbf{T}\,ds$ is positive. On the other hand, no vectors starting on C_2 point in the same direction as C_2, while some vectors point in roughly the opposite direction, so we would expect $\int_{C_2} \mathbf{F}\cdot d\mathbf{r} = \int_{C_2} \mathbf{F}\cdot\mathbf{T}\,ds$ to be negative.