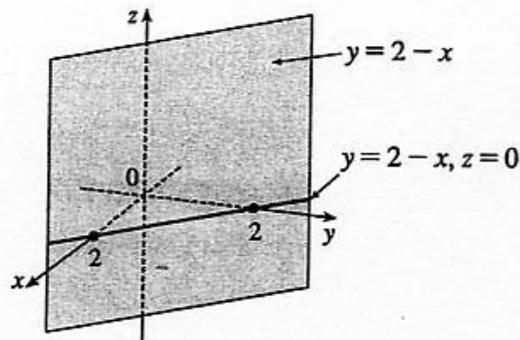


Homework 1a Math21a

- 5. The equation $x + y = 2$ represents the set of all points in \mathbb{R}^3 whose x - and y -coordinates have a sum of 2, or equivalently where $y = 2 - x$. This is the set $\{(x, 2 - x, z) \mid x \in \mathbb{R}, z \in \mathbb{R}\}$ which is a vertical plane that intersects the xy -plane in the line $y = 2 - x$, $z = 0$.



- 8. (a) The distance from a point to the xy -plane is the absolute value of the z -coordinate of the point. Thus, the distance is $|-5| = 5$.
- (b) Similarly, the distance is the absolute value of the x -coordinate of the point: $|3| = 3$.

(c) The distance is the absolute value of the y -coordinate of the point: $|7| = 7$.

(d) The point on the x -axis closest to $(3, 7, -5)$ is the point $(3, 0, 0)$. (Approach the x -axis perpendicularly.) The distance from $(3, 7, -5)$ to the x -axis is the distance between these two points:

$$\sqrt{(3-3)^2 + (7-0)^2 + (-5-0)^2} = \sqrt{74} \approx 8.60.$$

(e) The point on the y -axis closest to $(3, 7, -5)$ is $(0, 7, 0)$. The distance between these points is

$$\sqrt{(3-0)^2 + (7-7)^2 + (-5-0)^2} = \sqrt{34} \approx 5.83.$$

(f) The point on the z -axis closest to $(3, 7, -5)$ is $(0, 0, -5)$. The distance between these points is

$$\sqrt{(3-0)^2 + (7-0)^2 + [-5-(-5)]^2} = \sqrt{58} \approx 7.62.$$

- 10. An equation of the sphere with center $(6, 5, -2)$ and radius $\sqrt{7}$ is $(x-6)^2 + (y-5)^2 + [z-(-2)]^2 = (\sqrt{7})^2$ or $(x-6)^2 + (y-5)^2 + (z+2)^2 = 7$. The intersection of this sphere with the xy -plane is the set of points on the sphere whose z -coordinate is 0. Putting $z = 0$ into the equation, we have $(x-6)^2 + (y-5)^2 = 3, z = 0$ which represents a circle in the xy -plane with center $(6, 5, 0)$ and radius $\sqrt{3}$. To find the intersection with the xz -plane, we set $y = 0$: $(x-6)^2 + (z+2)^2 = -18$. Since no points satisfy this equation, the sphere does not intersect the xz -plane. (Also note that the distance from the center of the sphere to the xz -plane is greater than the radius of the sphere.) Similarly, the sphere does not intersect the yz -plane since substituting $x = 0$ into the equation gives $(y-5)^2 + (z+2)^2 = -29$.

- 14. Completing squares in the equation gives $4(x^2 - 2x + 1) + 4(y^2 + 4y + 4) + 4z^2 = 1 + 4 + 16 \Rightarrow 4(x-1)^2 + 4(y+2)^2 + 4z^2 = 21 \Rightarrow (x-1)^2 + (y+2)^2 + z^2 = \frac{21}{4}$, which we recognize as an equation of a sphere with center $(1, -2, 0)$ and radius $\sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$.

- 28. The equation $xyz = 0$ is satisfied when any of $x, y,$ or z is 0. Thus, the equation represents the region consisting of all points on the three coordinate planes $x = 0, y = 0,$ and $z = 0$.