

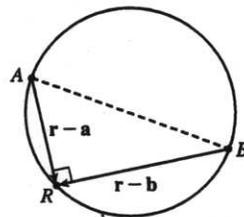
→ 20.  $|\langle -2, 4, 2 \rangle| = \sqrt{(-2)^2 + 4^2 + 2^2} = \sqrt{24} = 2\sqrt{6}$ , so a unit vector in the direction of  $\langle -2, 4, 2 \rangle$  is  $\mathbf{u} = \frac{1}{2\sqrt{6}} \langle -2, 4, 2 \rangle$ . A vector in the same direction but with length 6 is

$$6\mathbf{u} = 6 \cdot \frac{1}{2\sqrt{6}} \langle -2, 4, 2 \rangle = \left\langle -\frac{6}{\sqrt{6}}, \frac{12}{\sqrt{6}}, \frac{6}{\sqrt{6}} \right\rangle \text{ or } \langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle.$$

→ 34. Let  $P_1$  and  $P_2$  be the points with position vectors  $\mathbf{r}_1$  and  $\mathbf{r}_2$  respectively. Then  $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2|$  is the sum of the distances from  $(x, y)$  to  $P_1$  and  $P_2$ . Since this sum is constant, the set of points  $(x, y)$  represents an ellipse with foci  $P_1$  and  $P_2$ . The condition  $k > |\mathbf{r}_1 - \mathbf{r}_2|$  assures us that the ellipse is not degenerate.

→ 18.  $\langle -6, b, 2 \rangle$  and  $\langle b, b^2, b \rangle$  are orthogonal when  $\langle -6, b, 2 \rangle \cdot \langle b, b^2, b \rangle = 0 \Leftrightarrow (-6)(b) + (b)(b^2) + (2)(b) = 0$   
 $\Leftrightarrow b^3 - 4b = 0 \Leftrightarrow b(b+2)(b-2) = 0 \Leftrightarrow b = 0 \text{ or } b = \pm 2$ .

→ 34.  $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$  implies that the vectors  $\mathbf{r} - \mathbf{a}$  and  $\mathbf{r} - \mathbf{b}$  are orthogonal. From the diagram (in which  $A, B$  and  $R$  are the terminal points of the vectors), we see that this implies that  $R$  lies on a sphere whose diameter is the line from  $A$  to  $B$ . The center of this circle is the midpoint of  $AB$ , that is,



$\frac{1}{2}(\mathbf{a} + \mathbf{b}) = \left\langle \frac{1}{2}(a_1 + b_1), \frac{1}{2}(a_2 + b_2), \frac{1}{2}(a_3 + b_3) \right\rangle$ , and its radius is

$$\frac{1}{2}|\mathbf{a} - \mathbf{b}| = \frac{1}{2}\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}.$$

Or: Expand the given equation, substitute  $\mathbf{r} \cdot \mathbf{r} = x^2 + y^2 + z^2$  and complete the squares.

→ 16. The parallelogram is determined by the vectors  $\overrightarrow{KL} = \langle 0, 1, 3 \rangle$  and  $\overrightarrow{KN} = \langle 2, 5, 0 \rangle$ , so the area of parallelogram  $KLMN$  is

$$|\overrightarrow{KL} \times \overrightarrow{KN}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 2 & 5 & 0 \end{vmatrix} = |(-15)\mathbf{i} - (-6)\mathbf{j} + (-2)\mathbf{k}| = |-15\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}| = \sqrt{265} \approx 16.28.$$

→ 18. (a)  $\overrightarrow{PQ} = \langle 1, 1, 3 \rangle$  and  $\overrightarrow{PR} = \langle 3, 2, 5 \rangle$ , so a vector orthogonal to the plane through  $P, Q$ , and  $R$  is  $\overrightarrow{PQ} \times \overrightarrow{PR} = \langle (1)(5) - (3)(2), (3)(3) - (1)(5), (1)(2) - (1)(3) \rangle = \langle -1, 4, -1 \rangle$  (or any scalar multiple thereof).

→ 22.  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & 3 & -2 \\ 1 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 2 \begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} - 3 \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} + (-2) \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = -6 - 9 - 4 = -19$ . So the volume of the parallelepiped determined by  $\mathbf{a}, \mathbf{b}$  and  $\mathbf{c}$  is  $|-19| = 19$  cubic units.

→ 20.  $\mathbf{j} + 2\mathbf{k} = \langle 0, 1, 2 \rangle$  is a normal vector to the plane and  $\langle 4, 0, -3 \rangle$  is a point on the plane, so setting  $a = 0, b = 1, c = 2, x_0 = 4, y_0 = 0, z_0 = -3$  in Equation 6 gives  $0(x - 4) + 1(y - 0) + 2(z - (-3)) = 0$  or  $y + 2z = -6$  to be an equation of the plane.

→ 36. The plane will contain all perpendicular bisectors of the line segment joining the two points. Thus, a point in the plane is  $P_0 = \langle -1, -1, 2 \rangle$ , the midpoint of the line segment joining the two given points, and a normal to the plane is  $\mathbf{n} = \langle 6, -6, 2 \rangle$ , the vector connecting the two points. So an equation of the plane is  $6(x + 1) - 6(y + 1) + 2(z - 2) = 0$  or  $3x - 3y + z = 2$ .

→ 48. Put  $y = z = 0$  in the equation of the first plane to get the point  $\left(\frac{4}{3}, 0, 0\right)$  on the plane. Because the planes are parallel the distance  $D$  between them is the distance from  $\left(\frac{4}{3}, 0, 0\right)$  to the second plane. By Equation 8,

$$D = \frac{\left|1\left(\frac{4}{3}\right) + 2(0) - 3(0) - 1\right|}{\sqrt{1^2 + 2^2 + (-3)^2}} = \frac{1}{3\sqrt{14}}.$$