

6. $z = \frac{x}{y}, x = se^t, y = 1 + se^{-t} \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} = \frac{1}{y} (e^t) + \left(-\frac{x}{y^2}\right) (e^{-t}) = \frac{1}{y} e^t - \frac{x}{y^2} e^{-t}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} = \frac{1}{y} (se^t) + \left(-\frac{x}{y^2}\right) (-se^{-t}) = \frac{s}{y} e^t + \frac{xs}{y^2} e^{-t}$$

17. $z = y^2 \tan x, x = t^2 uv, y = u + tv^2 \Rightarrow$

$$\partial z / \partial t = (y^2 \sec^2 x) 2tuv + (2y \tan x) v^2, \partial z / \partial u = (y^2 \sec^2 x) t^2 v + 2y \tan x,$$

$$\partial z / \partial v = (y^2 \sec^2 x) t^2 u + (2y \tan x) 2tv. \text{ When } t = 2, u = 1 \text{ and } v = 0, \text{ we have } x = 0, y = 1, \text{ so } \partial z / \partial t = 0,$$

$$\partial z / \partial u = 0, \partial z / \partial v = 4.$$

18. $z = \frac{x}{y}, x = re^{st}, y = rse^t \Rightarrow$

$$\frac{\partial z}{\partial r} = \frac{1}{y} e^{st} + \frac{-x}{y^2} se^t, \frac{\partial z}{\partial s} = \frac{1}{y} rte^{st} - \frac{x}{y^2} re^t, \frac{\partial z}{\partial t} = \frac{1}{y} rse^{st} - \frac{x}{y^2} rse^t. \text{ When } r = 1, s = 2 \text{ and } t = 0, \text{ we}$$

$$\text{have } x = 1, y = 2, \text{ so } \partial z / \partial r = \frac{1}{2} + \frac{-1}{4} \cdot 2 = 0, \partial z / \partial s = 0 - \frac{1}{4} = -\frac{1}{4} \text{ and } \partial z / \partial t = \frac{1}{2} \cdot 2 - \frac{1}{4} \cdot 2 = \frac{1}{2}.$$

24. $xyz = \cos(x + y + z)$. Let $F(x, y, z) = xyz - \cos(x + y + z) = 0$, so

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{yz + \sin(x + y + z)}{xy + \sin(x + y + z)}, \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{xz + \sin(x + y + z)}{xy + \sin(x + y + z)}$$

28. (a) Since $\partial W / \partial T$ is negative, a rise in average temperature (while annual rainfall remains constant) causes a decrease in wheat production at the current production levels. Since $\partial W / \partial R$ is positive, an increase in annual rainfall (while the average temperature remains constant) causes an increase in wheat production.

- (b) Since the average temperature is rising at a rate of $0.15^\circ\text{C}/\text{year}$, we know that $dT/dt = 0.15$. Since rainfall is decreasing at a rate of $0.1 \text{ cm}/\text{year}$, we know $dR/dt = -0.1$. Then, by the Chain Rule,

$$\frac{dW}{dt} = \frac{\partial W}{\partial T} \frac{dT}{dt} + \frac{\partial W}{\partial R} \frac{dR}{dt} = (-2)(0.15) + (8)(-0.1) = -1.1. \text{ Thus we estimate that wheat production will decrease at a rate of } 1.1 \text{ units/year.}$$