10. By the Chain Rule (3), $\dfrac{\partial W}{\partial s} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial s} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial s}$. Then

$$W_s(1,0) = F_u\,(u(1,0), v(1,0))\,u_s(1,0) + F_v\,(u(1,0), v(1,0))\,v_s(1,0)$$
$$= F_u(2,3)u_s(1,0) + F_v(2,3)v_s(1,0) = (-1)(-2) + (10)(5) = 52$$

Similarly, $\dfrac{\partial W}{\partial t} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial t} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial t} \Rightarrow$

$$W_t(1,0) = F_u\,(u(1,0), v(1,0))\,u_t(1,0) + F_v\,(u(1,0), v(1,0))\,v_t(1,0)$$
$$= F_u(2,3)u_t(1,0) + F_v(2,3)v_t(1,0) = (-1)(6) + (10)(4) = 34$$

32. $I = \dfrac{V}{R} \Rightarrow \dfrac{dI}{dt} = \dfrac{\partial I}{\partial V}\dfrac{dV}{dt} + \dfrac{\partial I}{\partial R}\dfrac{dR}{dt} = \dfrac{1}{R}\dfrac{dV}{dt} - \dfrac{V}{R^2}\dfrac{dR}{dt} = \dfrac{1}{R}\dfrac{dV}{dt} - \dfrac{I}{R}\dfrac{dR}{dt}$

$$= \tfrac{1}{400}(-0.01) - \tfrac{0.08}{400}(0.03) = -0.000031 \text{ A/s}$$

43. $\dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}\cos\theta + \dfrac{\partial z}{\partial y}\sin\theta$ and $\dfrac{\partial z}{\partial \theta} = -\dfrac{\partial z}{\partial x}r\sin\theta + \dfrac{\partial z}{\partial y}r\cos\theta$. Then

$$\frac{\partial^2 z}{\partial r^2} = \cos\theta\left(\frac{\partial^2 z}{\partial x^2}\cos\theta + \frac{\partial^2 z}{\partial y\,\partial x}\sin\theta\right) + \sin\theta\left(\frac{\partial^2 z}{\partial y^2}\sin\theta + \frac{\partial^2 z}{\partial x\,\partial y}\cos\theta\right)$$
$$= \cos^2\theta\,\frac{\partial^2 z}{\partial x^2} + 2\cos\theta\sin\theta\,\frac{\partial^2 z}{\partial x\,\partial y} + \sin^2\theta\,\frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial^2 z}{\partial \theta^2} = -r\cos\theta\,\frac{\partial z}{\partial x} + (-r\sin\theta)\left(\frac{\partial^2 z}{\partial x^2}(-r\sin\theta) + \frac{\partial^2 z}{\partial y\,\partial x}r\cos\theta\right)$$
$$-r\sin\theta\,\frac{\partial z}{\partial y} + r\cos\theta\left(\frac{\partial^2 z}{\partial y^2}r\cos\theta + \frac{\partial^2 z}{\partial x\,\partial y}(-r\sin\theta)\right)$$
$$= -r\cos\theta\,\frac{\partial z}{\partial x} - r\sin\theta\,\frac{\partial z}{\partial y} + r^2\sin^2\theta\,\frac{\partial^2 z}{\partial x^2} - 2r^2\cos\theta\sin\theta\,\frac{\partial^2 z}{\partial x\,\partial y} + r^2\cos^2\theta\,\frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r} = (\cos^2\theta + \sin^2\theta)\frac{\partial^2 z}{\partial x^2} + (\sin^2\theta + \cos^2\theta)\frac{\partial^2 z}{\partial y^2} - \frac{1}{r}\cos\theta\,\frac{\partial z}{\partial x}$$
$$-\frac{1}{r}\sin\theta\,\frac{\partial z}{\partial y} + \frac{1}{r}\left(\cos\theta\,\frac{\partial z}{\partial x} + \sin\theta\,\frac{\partial z}{\partial y}\right)$$
$$= \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \text{ as}$$

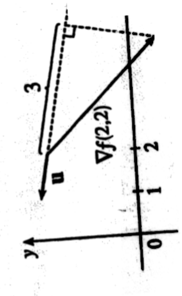
(a) $\nabla f(x,y) = \langle f_x(x,y), f_y(x,y)\rangle = \langle y/x, \ln x\rangle$ (b) $\nabla f(1,-3) = \langle \frac{-3}{1}, \ln 1\rangle = \langle -3, 0\rangle$

(c) By Equation 9, $D_{\mathbf{u}}f(1,-3) = \nabla f(1,-3)\cdot\mathbf{u} = \langle -3,0\rangle\cdot\langle -\frac{4}{5}, \frac{3}{5}\rangle = \frac{12}{5}$.

20. $f(x,y) = \ln(x^2 + y^2)$ $\nabla f(x,y) = \left\langle \dfrac{2x}{x^2+y^2}, \dfrac{2y}{x^2+y^2}\right\rangle$, $\nabla f(1,2) = \langle \frac{2}{5}, \frac{4}{5}\rangle$

change is $|\nabla f(1,2)| = \frac{2\sqrt{5}}{5}$ in the direction $\langle \frac{2}{5}, \frac{4}{5}\rangle$ or $\langle 2,4\rangle$.

16. $D_{\mathbf{u}}f(2,2) = \nabla f(2,2)\cdot\mathbf{u}$, the scalar projection of $\nabla f(2,2)$ onto $\mathbf{u}$, so we draw a perpendicular from the tip of $\nabla f(2,2)$ to the line containing $\mathbf{u}$. We can use the point $(2,2)$ to determine the scale of the axes, and we estimate the length of the projection to be approximately 3.0 units. Since the angle between $\nabla f(2,2)$ and $\mathbf{u}$ is greater than $90°$, the scalar projection is negative. Thus $D_{\mathbf{u}}f(2,2) \approx -3$.

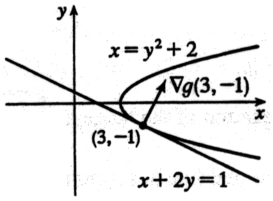


26. The fisherman is traveling in the direction $\langle -80, -60\rangle$. A unit vector in this direction is $\mathbf{u} = \frac{1}{100}\langle -80, -60\rangle = \langle -\frac{4}{5}, -\frac{3}{5}\rangle$, and if the depth of the lake is given by $f(x, y) = 200 + 0.02x^2 - 0.001y^3$, then $\nabla f(x, y) = \langle 0.04x, -0.003y^2\rangle$. $D_{\mathbf{u}} f(80, 60) = \nabla f(80, 60) \cdot \mathbf{u} = \langle 3.2, -10.8\rangle \cdot \langle -\frac{4}{5}, -\frac{3}{5}\rangle = 3.92$. Since $D_{\mathbf{u}} f(80, 60)$ is positive, the depth of the lake is increasing near $(80, 60)$ in the direction toward the buoy.

30. (a) Let $z = f(x, y) = 1000 - 0.01x^2 - 0.02y^2$. Then $\nabla f(x, y) = \langle -0.02x, -0.04y\rangle$. Proceed in the direction $\nabla f(60, 100) = \langle -1.2, -4\rangle$.

(b) The maximum slope is equal to the maximum directional derivative, which is $|\langle -1.2, -4\rangle| = \sqrt{17.44}$ and $\theta = \tan^{-1}\sqrt{17.44} \approx 76.5°$.

42. $\nabla g(x, y) = \langle 1, -2y\rangle$, $\nabla g(3, -1) = \langle 1, 2\rangle$. The tangent line has equation $\nabla g(3, -1) \cdot \langle x - 3, y + 1\rangle = 0 \;\Rightarrow$ $1(x - 3) + 2(y + 1) = 0$, which simplifies to $x + 2y = 1$.



43. $\nabla F(x_0, y_0, z_0) = \left\langle \frac{2x_0}{a^2}, \frac{2y_0}{b^2}, \frac{2z_0}{c^2}\right\rangle$. Thus an equation of the tangent plane at (x_0, y_0, z_0) is $\frac{2x_0}{a^2}x + \frac{2y_0}{b^2}y + \frac{2z_0}{c^2}z = 2\left(\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} + \frac{z_0^2}{c^2}\right) = 2(1) = 2$ since (x_0, y_0, z_0) is a point on the ellipsoid. Hence $\frac{x_0}{a^2}x + \frac{y_0}{b^2}y + \frac{z_0}{c^2}z = 1$ is an equation of the tangent plane.