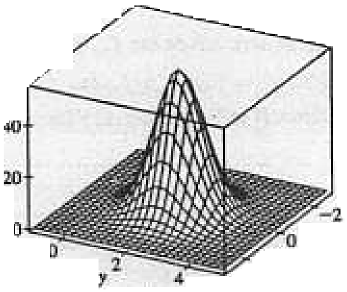
4. In the figure, points at approximately $(-1,1)$ and $(-1,-1)$ are enclosed by oval-shaped level curves which indicate that as we move away from either point in any direction, the values of f are increasing. Hence we would expect local minima at or near $(-1,\pm1)$. Similarly, the point $(1,0)$ appears to be enclosed by oval-shaped level curves which indicate that as we move away from the point in any direction the values of f are decreasing, so we should have a local maximum there. We also show hyperbola-shaped level curves near the points $(-1,0)$, $(1,1)$, and $(1,-1)$. The values of f increase along some paths leaving these points and decrease in others, so we should have a saddle point at each of these points.

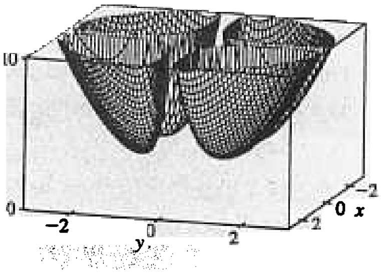
To confirm our predictions, we have $f(x,y) = 3x - x^3 - 2y^2 + y^4 \;\Rightarrow\; f_x(x,y) = 3 - 3x^2$, $f_y(x,y) = -4y + 4y^3$. Setting these partial derivatives equal to 0, we have $3 - 3x^2 = 0 \;\Rightarrow\; x = \pm1$ and $-4y + 4y^3 = 0 \;\Rightarrow\; y(y^2-1) = 0 \;\Rightarrow\; y = 0, \pm1$. So our critical points are $(\pm1, 0)$, $(\pm1,\pm1)$. The second partial derivatives are $f_{xx}(x,y) = -6x$, $f_{xy}(x,y) = 0$, and $f_{yy}(x,y) = 12y^2 - 4$, so $D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - [f_{xy}(x,y)]^2 = (-6x)(12y^2-4) - (0)^2 = -72xy^2 + 24x$. We use the Second Derivatives Test to classify the 6 critical points:

Critical Point	D	f_{xx}	Conclusion
$(1,0)$	24	-6	$D > 0,\ f_{xx} < 0 \;\Rightarrow\; f$ has a local maximum at $(1,0)$
$(1,1)$	-48		$D < 0 \;\Rightarrow\; f$ has a saddle point at $(1,1)$
$(1,-1)$	-48		$D < 0 \;\Rightarrow\; f$ has a saddle point at $(1,-1)$
$(-1,0)$	-24		$D < 0 \;\Rightarrow\; f$ has a saddle point at $(-1,0)$
$(-1,1)$	48	6	$D > 0,\ f_{xx} > 0 \;\Rightarrow\; f$ has a local minimum at $(-1,1)$
$(-1,-1)$	48	6	$D > 0,\ f_{xx} > 0 \;\Rightarrow\; f$ has a local minimum at $(-1,-1)$

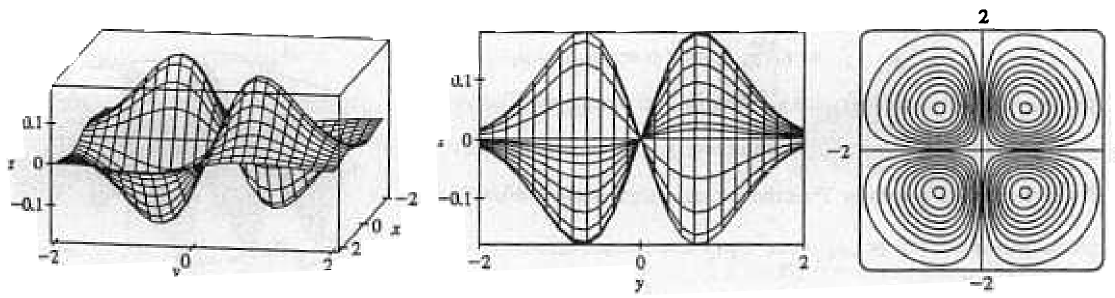
8. $f(x,y) = e^{4y-x^2-y^2} \;\Rightarrow\; f_x = -2xe^{4y-x^2-y^2}$, $f_y = (4-2y)e^{4y-x^2-y^2}$, $f_{xx} = (4x^2-2)e^{4y-x^2-y^2}$, $f_{xy} = -2x(4-2y)e^{4y-x^2-y^2}$, $f_{yy} = (4y^2 - 16y + 14)e^{4y-x^2-y^2}$. Then $f_x = 0$ and $f_y = 0$ implies $x = 0$ and $y = 2$, so the only critical point is $(0,2)$. $D(0,2) = (-2e^4)(-2e^4) - 0^2 = 4e^8 > 0$ and $f_{xx}(0,2) = -2e^4 < 0$, so $f(0,2) = e^4$ is a local maximum.



12. $f(x,y) = x^2 + y^2 + \dfrac{1}{x^2y^2} \;\Rightarrow\; f_x = 2x - 2x^{-3}y^{-2}$, $f_y = 2y - 2x^{-2}y^{-3}$, $f_{xx} = 2 + 6x^{-4}y^{-2}$, $f_{yy} = 2 + 6x^{-2}y^{-4}$, $f_{xy} = 4x^{-3}y^{-3}$. Then $f_x = 0$ implies $2x^4y^2 - 2 = 0$ or $x^4y^2 = 1$ or $y^2 = x^{-4}$. Note that neither x nor y can be zero. Now $f_y = 0$ implies $2x^2y^4 - 2 = 0$, and with $y^2 = x^{-4}$ this implies $2x^{-6} - 2 = 0$ or $x^6 = 1$. Thus $x = \pm1$ and if $x = 1$, $y = \pm1$; if $x = -1$, $y = \pm1$. So the critical points are $(1,1)$, $(1,-1)$, $(-1,1)$ and $(-1,-1)$. Now $D(\pm1,\pm1) = D(\pm1,\mp1) = 64 - 16 > 0$ and $f_{xx} > 0$ always, so $f(\pm1,\pm1) = f(\pm1,\mp1) = 3$ are local minima.



16. $f(x, y) = xye^{-x^2-y^2}$



There appear to be local maxima of about $f(\pm 0.7, \pm 0.7) \approx 0.18$ and local minima of about $f(\pm 0.7, \mp 0.7) \approx -0.18$. Also, there seems to be a saddle point at the origin.
$f_x = ye^{-x^2-y^2}(1 - 2x^2)$, $f_y = xe^{-x^2-y^2}(1 - 2y^2)$, $f_{xx} = 2xye^{-x^2-y^2}(2x^2 - 3)$,
$f_{yy} = 2xye^{-x^2-y^2}(2y^2 - 3)$, $f_{xy} = (1 - 2x^2)e^{-x^2-y^2}(1 - 2y^2)$. Then $f_x = 0$ implies $y = 0$ or $x = \pm\frac{1}{\sqrt{2}}$.
Substituting these values into $f_y = 0$ gives the critical points $(0, 0)$, $\left(\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$. Then
$D(x, y) = e^{2(-x^2-y^2)}\left[4x^2y^2(2x^2 - 3)(2y^2 - 3) - (1 - 2x^2)^2(1 - 2y^2)^2\right]$, so $D(0, 0) = -1$, while
$D\left(\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) > 0$ and $D\left(-\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) > 0$. But $f_{xx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) < 0$, $f_{xx}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) > 0$,
$f_{xx}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) > 0$ and $f_{xx}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) < 0$. Hence $(0, 0)$ is a saddle point;
$f\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = f\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -\frac{1}{2e}$ are local minima and $f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = \frac{1}{2e}$ are local maxima.

26. $f_x(x, y) = 4 - 2x$ and $f_y(x, y) = 6 - 2y$, so the only critical point is $(2, 3)$ (which is in D) where $f(2, 3) = 13$.
Along L_1: $y = 0$, so $f(x, 0) = 4x - x^2 = -(x - 2)^2 + 4$, $0 \le x \le 4$, which has a maximum value when $x = 2$ where $f(2, 0) = 4$ and a minimum value both when $x = 0$ and $x = 4$, where $f(0, 0) = f(4, 0) = 0$. Along L_2: $x = 4$, so $f(4, y) = 6y - y^2 = -(y - 3)^2 + 9$, $0 \le y \le 5$, which has a maximum value when $y = 3$ where $f(4, 3) = 9$ and a minimum value when $y = 0$ where $f(4, 0) = 0$. Along L_3: $y = 5$, so $f(x, 5) = -x^2 + 4x + 5 = -(x - 2)^2 + 9$, $0 \le x \le 4$, which has a maximum value when $x = 2$ where $f(2, 5) = 9$ and a minimum value both when $x = 0$ and $x = 4$, where $f(0, 5) = f(4, 5) = 5$. Along L_4: $x = 0$, so $f(0, y) = 6y - y^2 = -(y - 3)^2 + 9$, $0 \le y \le 5$, which has a maximum value when $y = 3$ where $f(0, 3) = 9$ and a minimum value when $y = 0$ where $f(0, 0) = 0$. Thus the absolute maximum is $f(2, 3) = 13$ and the absolute minimum is attained at both $(0, 0)$ and $(4, 0)$, where $f(0, 0) = f(4, 0) = 0$.