10/16/2002 FIRST HOURLY **Math 21a**

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- Start by printing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		80
2		30
3		30
4		40
5		30
6		30
7		30
8		40
9		40
Total:		350

Problem 1) (80 points)

Circle for each of the 20 questions the correct letter. No justifications are needed.

T	F	The vector connecting the point $(1, 4, 2)$ with the point $(1, 1, 1)$ is parallel to the vector $\langle 0, -6, -2 \rangle$.
T	F	The length of the sum of two vectors is the sum of the length of the vectors.
T	F	For any three vectors, $\vec{v} \cdot (\vec{w} + \vec{u}) = \vec{w} \cdot \vec{v} + \vec{u} \cdot \vec{v}$.
T	F	For any three vectors, $(\vec{v} \times \vec{w}) \cdot \vec{u} = (\vec{w} \times \vec{v}) \cdot \vec{u}$.
T	F	For any three vectors $\lvert(\vec{u} \times \vec{v}) \cdot \vec{w}\rvert = \lvert(\vec{u} \times \vec{w}) \cdot \vec{v}\rvert$.
T	F	The vectors $\vec{i} + \vec{j}$ and $\vec{k}$ are orthogonal.
T	F	For any vector $\vec{v}$ one has $\vec{v} \times (2\vec{v}) = 0$.
T	F	If we attach the vector $\langle 2, 1, 1 \rangle$ to the point $P = (2, 3, 4)$, the tip of the vector points to the point $Q = (3, 4, 5)$.
T	F	The set of points which have distance 1 from a plane form a single plane.
T	F	$\lvert\vec{v} \times \vec{w}\rvert = 0$ implies $\vec{v} = 0$ or $\vec{w} = 0$.
T	F	The set of points which satisfy $x^2 + 2x + y^2 - z^2 = 0$ is a cone.
T	F	If $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal, then the vectors $\vec{u}$ and $\vec{v}$ have the same length.
T	F	If P, Q, R are 3 different points in space that don't lie in a line, then $\vec{PQ} \times \vec{RQ}$ is a vector orthogonal to the plane containing P, Q, R.
T	F	The line $\vec{r}(t) = (1+2t, 1+3t, 1+4t)$ hits the plane $2x+3y+4z = 9$ at a right angle.
T	F	If in rectangular coordinates, a point is given by $(1, -1, 0)$, then its spherical coordinates are $(\rho, \theta, \phi) = (\sqrt{2}, -\pi/2, \pi/2)$.
T	F	If the velocity vector of the curve $\vec{r}(t)$ is never zero and always parallel to a constant vector $\vec{v}$ for all times t, then the curve is a straight line.
T	F	The equation $r = 3z$ in cylindrical coordinates defines a cone.
T	F	The set of points in the $x - y$ plane which satisfy $x^2 - 2y^2 = 0$ is an ellipse.
T	F	A surface which is given as $r = \sin(z)$ in cylindrical coordinates stays the same when we rotate it around the y axes.
T	F	The identity $\lvert\vec{v} \cdot \vec{w}\rvert^2 + \lvert\vec{v} \times \vec{w}\rvert^2 = \lvert\vec{v}\rvert^2 \lvert\vec{w}\rvert^2$ holds for all vectors $\vec{v}, \vec{w}$.

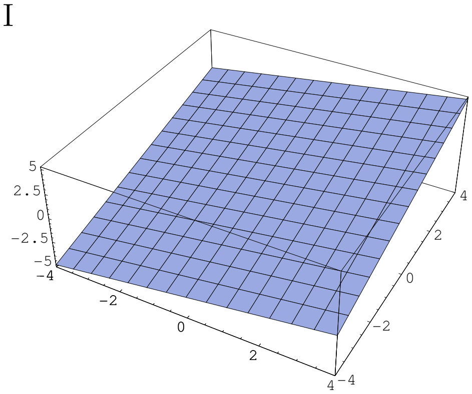
	x 4 =	

Space for work

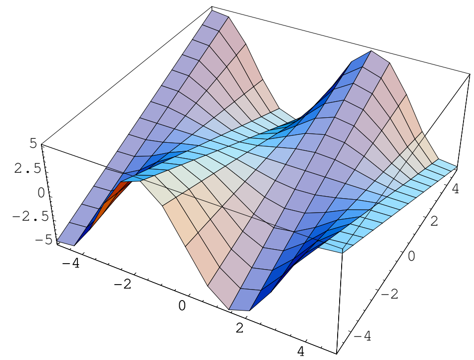
Problem 2) (30 points)

Match the equation with their graphs. To do so, it can help to look at the intersection of each surface with the xy-plane.

I



II



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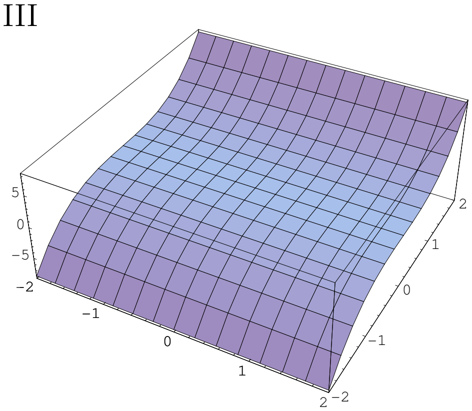

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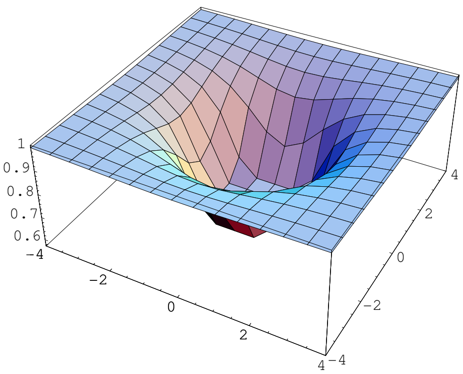

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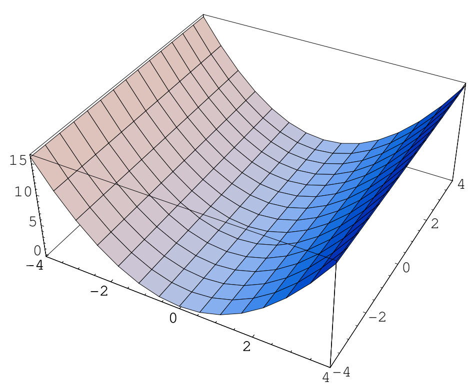
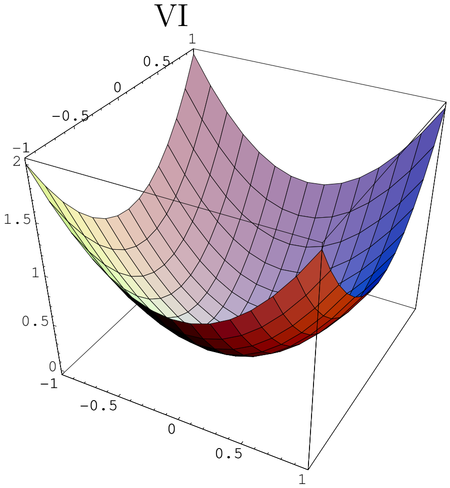


VI



I,II,III,IV,V or VI?	Equation
	$z = \sin(x)y$
	$z = \cos(\frac{\pi}{(1+x^2+y^2)})$
	$2x + 3y + 4z = 0$

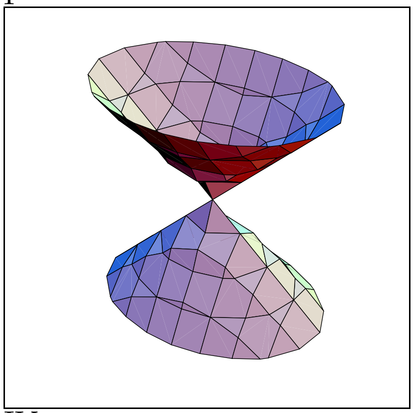
I,II,III,IV,V or VI?	Equation
	$z = y^3$
	$z = x^2$
	$z = x^2 + y^2$

Space for work

Problem 3) (30 points)

Match the equation with their graphs and describe the x-y trace (the intersection of the surface with the xy-plane) with at most three words in each case.

I



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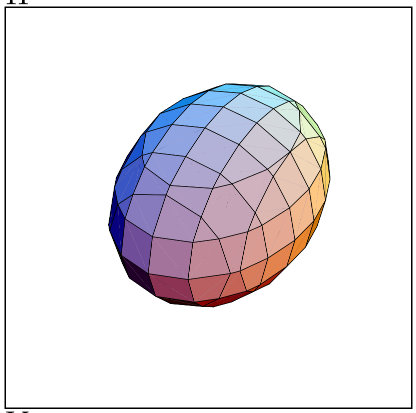

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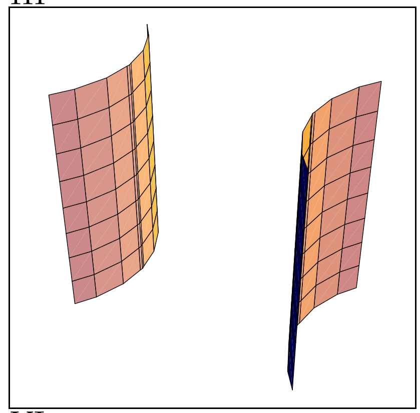

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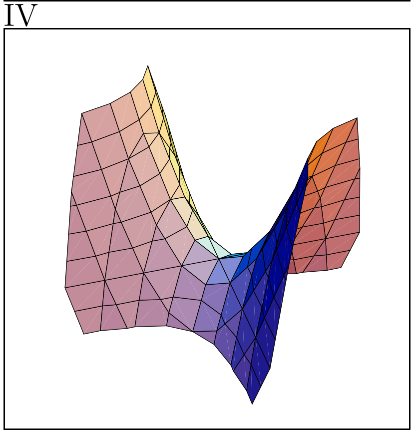

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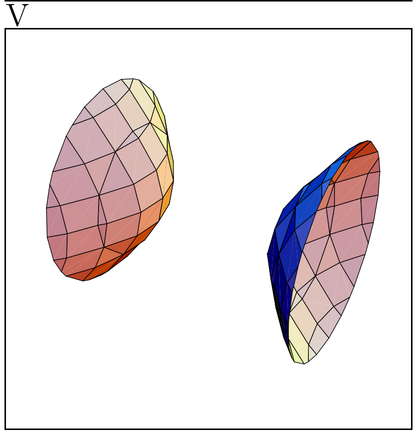
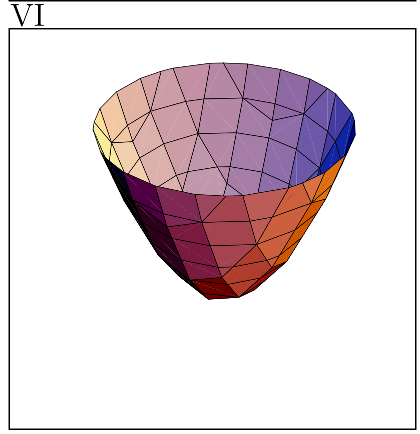


VI



Enter I,II,III,IV,V,VI here	Equation	Describe the x-y trace in words
	$x^2 - y^2 - z^2 = 1$	
	$x^2 + 2y^2 = z^2$	
	$2x^2 + y^2 + 2z^2 = 1$	
	$x^2 - y^2 = 5$	
	$x^2 - y^2 - z = 1$	
	$x^2 + y^2 - z = 1$	

Space for work

Problem 4) (40 points)

Find the distance between the point $P = (1, 0, -1)$ and the plane which contains the points $A = (1, 1, 1)$ and $B = (0, 2, 1)$ and $C = (1, 2, 2)$.
To do so:

a) Find an equation of the plane.

b) Find the distance.

Space for work

Problem 5) (30 points)

An ant has gotten into the math department 'surface cabinet' and is walking around on one of the models. Her position in cylindrical coordinates is

$$
\begin{aligned}
r(t) &= 2\sqrt{t} \\
\theta(t) &= t \\
z(t) &= 2t
\end{aligned}
$$

for $0 \leq t \leq 6\pi$.

a) What are the parametric equations describing the ant's path in rectangular coordinates?

b) Write an equation (in either rectangular or cylindrical coordinates) which might describe the surface the ant is walking on. Sketch the surface given by your equation, and indicate the ant's path.
(Hint: there are many possible surfaces. You may find some easier to draw than others.)

Space for work

Problem 6) (30 points)

In the xy-plane, the equation $x^2 - y^2 = 1$ defines a hyperbola. If we rotate this hyperbola around the y-axis, we obtain a surface in three dimensional space. Sketch this surface and write an equation which defines it.

Space for work

Problem 7) (30 points)

Let $\vec{a}$ and $\vec{b}$ be two vectors in $\mathbf{R}^3$. Assume that the length of $\vec{a} \times \vec{b}$ is equal to 10. What is the length of $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$?

Space for work

Problem 8) (40 points)

Consider the parametrized curve $\vec{r}(t) = (e^t \cos(t), e^t \sin(t), e^t)$.

a) Find a parametric equation for the tangent line to this curve at $t = \pi$.

b) Find a scalar equation for the plane perpendicular to the curve at the same point.

c) Find the arclength of the segment of the curve for which $0 \leq t \leq 1$.

Space for work

Problem 9) (40 points)

Let C be the curve of the intersection of the elliptical cylinder $\frac{x^2}{25} + \frac{y^2}{9} = 1$ in three dimensions with the plane $3z = 4y$.

a) Find a parametric equation $\vec{r}(t) = (x(t), y(t), z(t))$ of C.

b) Find the arc length of C.