SOLUTIONS TO PRACTICE EXAM FIRST HOURLY **Math 21a**

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- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		80
2		30
3		30
4		30
5		30
6		30
7		30
8		30
9		30
10		30
Total:		350

Problem 1) TF questions (80 points)

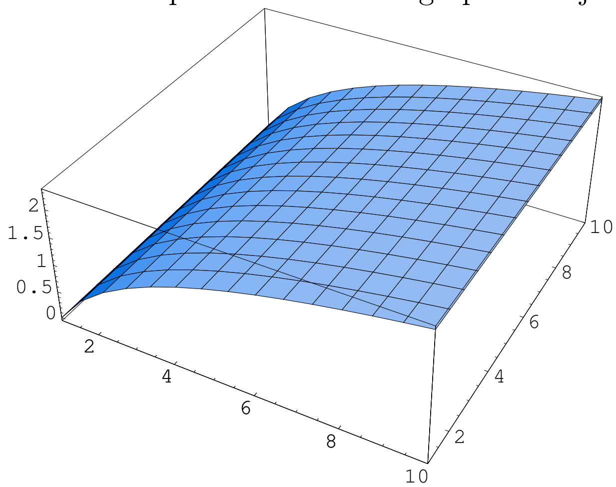
Circle for each of the 20 questions the correct letter. No justifications are needed. A (*) indicates the correct solution.

T	*	The vectors $\langle 3,-2,1 \rangle$ and $\langle -6,4,2 \rangle$ are parallel.
T	*	The length of the vector $\langle 3,4,0 \rangle$ is 25.
T	*	For any two vectors, $v \times w = w \times v$.
*	F	The vectors $\langle 1,1 \rangle$ and $\langle 1,-1 \rangle$ are orthogonal.
T	*	For any two vectors v, w one has $\|\|v+w\|\|^2 = \|\|v\|\|^2 + \|\|w\|\|^2$.
*	F	The surface $x^2 - y^2 + z^2 = 1$ is a one-sheeted hyperboloid.
*	F	The set of points which have distance 1 from a line is a cylinder.
*	F	If $\|v \times w\| = 0$ for all vectors w, then $v = 0$.
T	*	The intersection of two planes is always a line.
*	F	If u and u are orthogonal vectors, then $(u \times v) \times u$ is parallel to v.
*	F	Two vectors are parallel if and only if their cross product vanishes.
T	*	Two general lines in three dimensional space always intersect in a point.
T	*	Every vector contained in the line $r(t) = (1+2t, 1+3t, 1+4t)$ is parallel to the vector $(1,1,1)$.
*	F	If in spherical coordinates a point is given by $(\rho,\theta,\phi) = (2,\pi/2,\pi/2)$, then its rectangular coordinates are $(x,y,z) = (0,2,0)$.
*	F	If the velocity vector $r'(t)$ of the planar curve $r(t)$ is orthogonal to the vector $r(t)$ for all times t, then the curve is a circle.
T	*	Every point on the sphere of radius ρ is determined alone by its angle ϕ from the z axes.
T	*	The equation $r = 3$ in cylindrical coordinates is a sphere.
T	*	The set of points which satisfy $x^2 - 2y^2 - 3z^2 = 0$ are on an ellipsoid.
*	F	A surface which is given as $r = 2 + \sin(z)$ in cylindrical coordinates stays the same when we rotate it around the z axes.
T	*	If $v \times w = (0,0,0)$, then $v = w$.

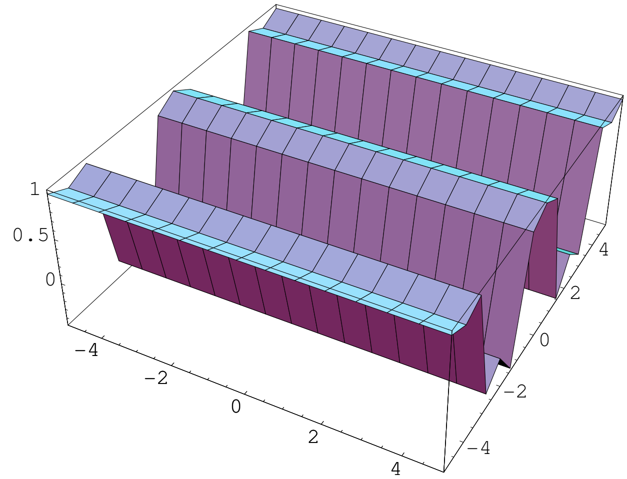
[ ] x 4 = [ ]

Problem 2) (30 points)

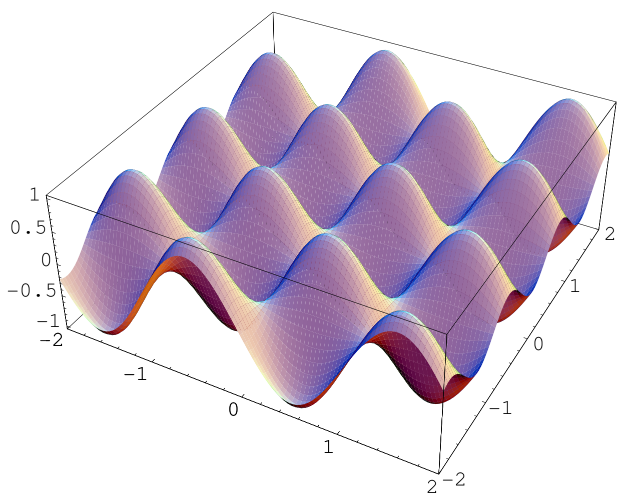
Match the equation with their graphs and justify briefly your choice.



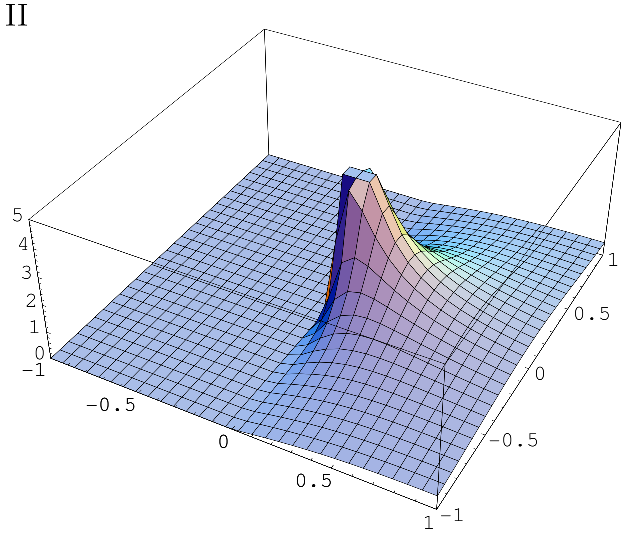
I



II



III

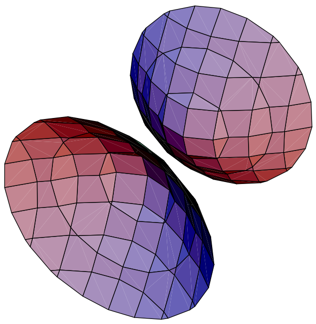


IV

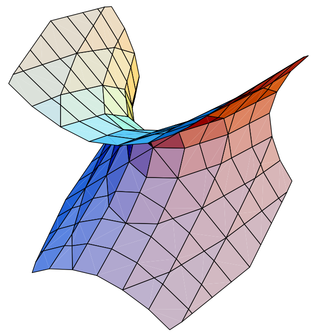
Enter I,II,III,IV here	Equation	Short Justification
III	$z = \sin(3x)\cos(5y)$	two traces show waves
II	$z = \cos(y^2)$	no x dependence, periodic in y
I	$z = \log(x)$	no y dependence, monotone in x
IV	$x/(x^2 + y^2)$	singular at (x,y)=(0,0)

Problem 3) (30 points)

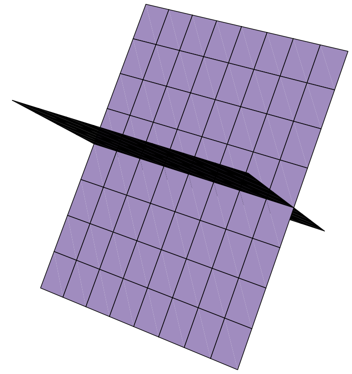
Match the equation with their graphs and justify briefly your choice.



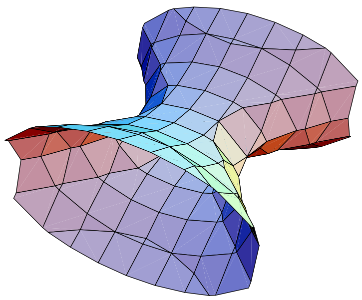
I



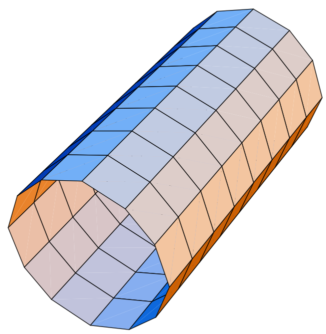
II



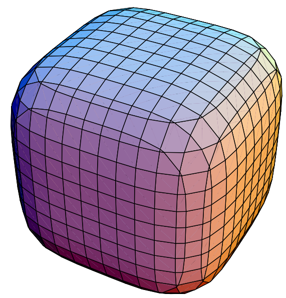
III



IV



V



VI

Enter I,II,III,IV,V,VI here	Equation	Short explanation
VI	$x^4 + y^4 + z^4 - 1 = 0$	all three traces are circle like
I	$-x^2 + y^2 - z^2 - 1 = 0$	x-y trace is hyperbola, xz trace is empty
V	$x^2 + z^2 = 1$	no y dependence: cylinder
III	$-y^2 + z^2 = 0$	$y^2 = z^2$ means y=z or y=-z, two planes
IV	$x^2 - y^2 + 3z^2 - 1 = 0$	x z trace is ellipse, xy trace is hyperbola
II	$x^2 - y - z^2 = 0$	parabolas and hyperbola appear as traces

Problem 4) (30 points)

Given the vectors $v = \langle 1,1,0 \rangle$ and $w = \langle 0,0,1 \rangle$ and the point $P = (2,4,-2)$. Let Σ be the plane which goes through the origin and contains the vectors v and w.
a) Determine the distance from P to the origin.

Solution: $\sqrt{4+16+4} = \sqrt{24} = 2\sqrt{6}$.

b) Determine the distance from P to the plane Σ.

Solution: $\Sigma : x - y = 0$, $n = \langle 1,-1,0 \rangle$. $Q = (0,0,0)$ is a point on the plane. $\vec{PQ} \cdot n/||n|| = \langle 2,4,-2 \rangle \cdot \langle 1,-1,0 \rangle/\sqrt{2} = 2/\sqrt{2} = \sqrt{2}$

Problem 5) (30 points)

Boba Fett is flying through the air when his rocket pack malfunctions and sends him spinning out of control. At time $t = 0$, he is at the point $P_0 = (0,0,27)$ and moving with velocity $\vec{v} = \langle 10,0,0 \rangle$. While he is in the air, his acceleration is given by $\vec{a}(t) = \langle \pi^2 \sin \pi t, \pi^2 \cos \pi t + 2t, -6t \rangle$ for $t \geq 0$.

1. For $t \geq 0$, find Boba's position as a function of time.

   Solution: Integrate $r''(t)$ and compare $r'(0) = (10,0,0)$, then integrate again and compare with $r(0) = (0,0,27)$. Result: $r(t) = (10t + \pi t - \sin(\pi t), -\cos(\pi t) + t^3/3 + 1, 27 - t^3)$.

2. The ground is represented by the xy plane. At what time does Boba hit the ground? What are the x and y coordinates of the point, where he hits?

   Solution: $27 - t^3 = 0$ means $t = 3$. At that time $r(3) = (30 + 3\pi, 11, 0)$.

Problem 6) (30 points)

Let $P = (1,-1,4)$ and $Q = (-7,9,0)$ be points in space, and let M be the plane given by the equation $x - 2y + 2z = -7$.

1. Find a vector equation for the line L passing through P and Q.

   Solution: $r(t) = P + t\vec{PQ} = (1,-1,4) + t(-8,10,-4) = (x(t), y(t), z(t))$

2. Find the point R at which L intersects M.

   Solution: $x - 2y + 2z = (1-8t) - 2(-1+10t) + 2(4-4t) = 11 - 36t$. If $11 - 36t = -7$, then $((x(t), y(t), z(t))$ is on the plane. $t = 1/2$. Then $r(1/2) = (1,-1,4) + (-4,5,-2) = (-3,4,2)$.

3. Let S be the projection of P to M (so that PS is perpendicular to M.) Find the area of the triangle $\triangle PRS$. (Hint: first find the vector PS.)

Solution: The distance of P to the plane is $d = |\vec{PR} \cdot n|/||n|| = |(4, -5, 2) \cdot (1, -2, 2)|/3 = 6$. The vector $\vec{PS}$ is therefore $6n/||n|| = 2n = (2, -4, 4)$. The area of the triangle is $||\vec{PS} \times \vec{PR}||/2 = ||(-2, 4, -4) \times (4, -5, 2)||/2 = ||(-12, -12, -6)||/2 = 9$.

Problem 7) (30 points)

a) Identify the surface whose equation is given in spherical coordinates as $\theta = \pi/4$.

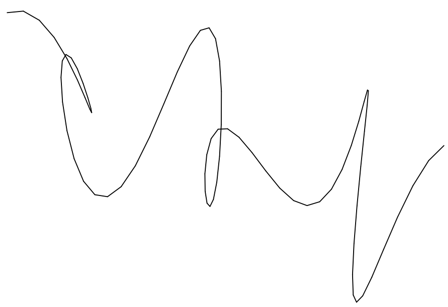
Solution: A half plane contained in $x = y$.

b) Identify the surface whose equation is given in spherical coordinates as $\phi = \pi/4$.
Solution: A half cone. $x^2 + y^2 = z^2$ with $z \geq 0$.

c) Identify the surface, whose equation is given in cylindrical coordinates by $z = r^2$. Either name it or sketch the surface convincingly.

Solution: $z = x^2 + y^2$ is a circular paraboloid.

Problem 8) (30 points)



Let $r(t)$ be the space curve $r(t) = (t^2, \sin(3\pi t), \cos(5\pi t))$.

a) Calculate the velocity vector and the acceleration of $r(t)$ at time $t = 1$.

$v(t) = r'(t) = (2t, 3\pi\cos(3\pi t), -5\pi\sin(5\pi t))$. $v(1) = (2, -3\pi, 0)$ $a(t) = r''(t) = (2, -9\pi^2\sin(3\pi t), -25\pi^2$ cos
$a(1) = (2, 0, 25\pi^2)$.

b) Calculate the speed of $r(t)$ at time $t = 1$.

Solution: $||v(1)|| = \sqrt{4 + 9\pi^2}$.
c) Write down the length of the curve from $t = 1$ to $t = 10$ as integral. You don't have to evaluate the integral.
$\int_1^1 0\sqrt{4t^2 + 9\pi^2\cos^2(3\pi t) + 25\pi^2\sin^2(5\pi t)}$.

Problem 9) (30 points)

Let S be the surface given by

$$z^2 = \frac{x^2}{4} + y^2 \ .$$

a) Sketch the surface S.

Solution: It is an elliptical cone. Every z-trace is an ellipse.
b) Let (a, b, c) be a point on the surface S. Find a parametric equation for the line that passes through (a, b, c) and lies entirely on the surface S.

Solution: The line $r(t) = (at, bt, ct)$ lies on the surface.

Problem 10) (30 points)

The curve $t \mapsto \vec{r}(t) = (t^3, 1 - t, 1 - t^3)$ lies in a plane. What is the equation of this plane?
Solution: $t = 0 : P = (0, 1, 1), t = 1 : Q = (1, 0, 0), t = 2 : R = (8, -1, -7)$ are points on the Plane. $\vec{PQ} = (1, -1, -1)$, $\vec{PR} = (8, -2, -8)$. Their cross product is $(6, 0, 6)$. The plane is $x + z = 1$.