11/21/2002 SECOND HOURLY, PRACTICE EXAM **Math 21a**

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- Start by printing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit. Justify your answers.
- No notes, books, calculators, computers or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		80
2		30
3		40
4		40
5		40
6		40
7		40
8		40
Total:		350

Problem 1) TF questions (80 points) Circle the correct letter. No justifications are needed.

T	F	At a local maximum (x_0, y_0) of $f(x, y)$, one has $f_{yy}(x_0, y_0) \geq 0$.
T	F	If R is the region bounded by $x^2 + 4y^2 = 1$ then $\int \int_R xy^4 \, dxdy < 0$.
T	F	The gradient $\langle 2x, 2y \rangle$ is perpendicular to the surface $z = x^2 + y^2$.
T	F	The equation $f(x, y) = k$ implicitly defines x as a function of y and $\frac{dx}{dy} = \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}$.
T	F	$f(x, y) = \sqrt{(16 - x^2 - y^2)}$ has both an absolute maximum and an absolute minimum on its domain of definition.
T	F	If (x_0, y_0) is a critical point of $f(x, y)$ under the constraint $g(x, y) = 0$, and $f_{xy}(x_0, y_0) < 0$, then (x_0, y_0) is a saddle point.
T	F	The vector $r_u(u, v)$ of a parameterized surface $(u, v) \mapsto r(u, v) = (x(u, v), y(u, v), z(u, v))$ is normal to the surface.
T	F	The identity $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) \, dydx = \int_0^1 \int_0^{\pi/2} r^2 \, d\theta dr$ holds.
T	F	$f(x, y)$ and $g(x, y) = f(x^2, y^2)$ have the same critical points.
T	F	If $f(x, t)$ satisfies the Laplace equation $f_{xx} + f_{tt} = 0$ and simultaneously the wave equation $f_{xx} = f_{tt}$, then $f(x, t) = ax + bt + c$.
T	F	Every smooth function satisfies the partial differential equation $f_{xxyy} = f_{xyxy}$.
T	F	The function $(x^4 - y^4)$ has has neither a local maximum nor a local minimum at $(0, 0)$.
T	F	$\int_0^1 \int_0^{\pi/2} r \, d\theta dr = \pi/4$.
T	F	At a saddle point, the directional derivative is zero for two different vectors u, v.
T	F	It is possible to find a function of two variables which has no maximum and no minimum.
T	F	The value of the function $f(x, y) = e^x y$ at $(0.001, -0.001)$ can by linear approximation be estimated as -0.001.
T	F	For any function $f(x, y, z)$ and any unit vectors u, v, one has the identity $D_{u \times v} f(x, y, z) = D_u f(x, y, z) D_v f(x, y, z)$.
T	F	Given 2 arbitrary points in the plane, there is a function $f(x, y)$ which has these points as critical points and no other critical points.
T	F	The maximum of $f(x, y)$ under the constraint $g(x, y) = 0$ is the same as the maximum of $g(x, y)$ under the constraint $f(x, y) = 0$.
T	F	Assume (x_0, y_0) is a critical point of $f(x, y)$ and $f_{xx} f_{yy} - f_{xy}^2 \neq 0$ at this point. Let T be the tangent plane of the surface $S = \{f(x, y) - z = 0\}$ at $P = (x_0, y_0, f(x_0, y_0))$. If the intersection of T with S is a single point, then (x_0, y_0) is a local max or local min.

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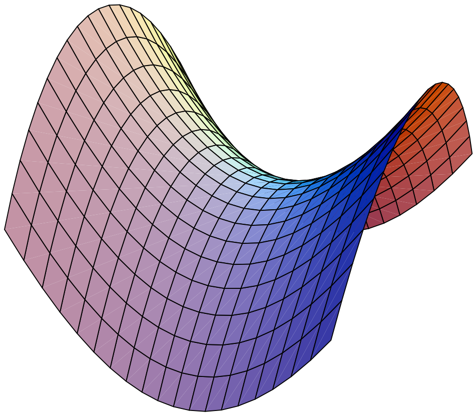
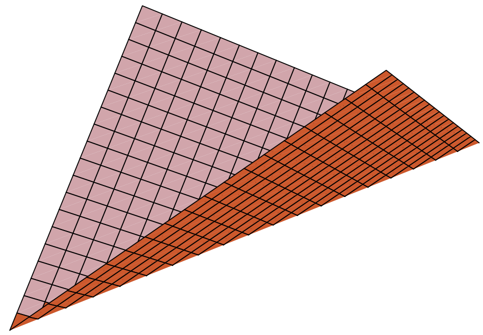
Problem 2) (30 points)

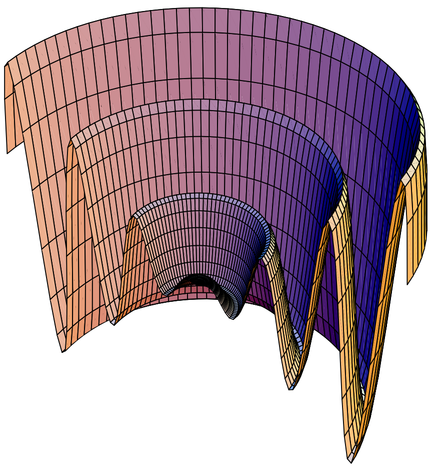
Match the parametric surfaces with their parameterization. No justification is needed.



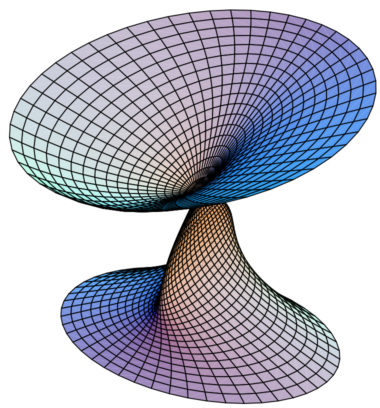
I



II



III



IV

Enter I,II,III,IV here	Parameterization
	$(u, v) \mapsto (u\cos(v), u\sin(v), u^2\cos(u)/(u+1))$
	$(u, v) \mapsto (u, v+u, \lvert u-v \rvert)$
	$(u, v) \mapsto ((u-\sin(u))\cos(v), (u-\cos(u))\sin(v), u)$
	$(u, v) \mapsto (u, v, u^2-v^2)$

Problem 3) (40 points)

Find all the critical points of the function $f(x, y) = xy(4-x^2-y^2)$. Are they maxima, minima or saddle points?

Problem 4) (40 points)

Let $f(x, y) = e^{(x-y)}$ so that $f(\log(2), \log(2)) = 1$. Find the equation for the tangent plane to the graph of f at $(\log(2), \log(2))$ and use it to estimate $f(\log(2) + 0.1, \log(2) + 0.04)$.

Problem 5) (40 points)

f is a function which depends on x and y, where $x = u^3 v$ and $y = u^2 v^2$. When $(u, v) = (1, 1)$ $\frac{\partial f}{\partial x} = -5$, $\frac{\partial f}{\partial v} = 9$. What is $\frac{\partial f}{\partial u}$?

Problem 6) (40 points)

A **can** is a cylinder with a circular base. Its surface area (top, bottom and sides) is 300π cm^2. What is the maximum possible volume of such a can?

Problem 7) (40 points)

Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{xy^5}{x^2+y^2}\, dy\, dx$.

Problem 8) (40 points)

a) Find the area of the region D enclosed by the lines $x = \pm 2$ and the parabolas $y = 1 + x^2$, $y = -1 - x^2$.

b) Find the integral of $f(x, y) = y^2$ on the same region as in a). (The result can be interpreted as a moment of inertia).