ANSWERS:

1. a) $x - (2 - 2y + y^2)^{1/4} = 0$
   b) $(x^2 + 4)^{1/2} - 2y = 0$.
   c) $x^2/16 + y^2/9 = 1$.

2. a) $x = 1$.
   b) $27x + 4y = -73$.
   c) $x + 3y = -1$.

3. $e^2 + 1$.

4. a) $(t\cos(t^3), t\sin(t^3))$.
   b) $x\sin((x^2 + y^2)^{3/2}) - y\cos((x^2 + y^2)^{3/2}) = 0$.

5. a) $\mathbf{B} = 1/3\, \mathbf{A} + 1/3\, (2, 4, -5)$.
   b) $\mathbf{B} = 11/25\, \mathbf{A} + 1/25\, (92, 69, 25)$.
   c) $\mathbf{B} = -1/3\, \mathbf{A} + 1/3\, (11, 8, 7)$.

6. Only in Case c) are **v** and **w** perpendicular.

7. a) 2/3.
   b) $(13)^{1/2}/7$.

8. a) $2x + y - z = 2$.
   b) $x + y - z = 3$.
   c) $x - y = -1$.

9. 6/7.

10. $t \to (t, t, t)$ or $t \to (-t, -t, -t)$.

11. a) Yes. b) No. c) Yes. The answer is yes if there is a constant, non-zero vector which is orthogonal to **v** at each time t. Otherwise, the answer is no. For a), consider (-1, 0, 5) and for c), consider (0, 7, 1). No such vector exists for b) since in this case, $\mathbf{v}(0) = (0, 0, 1)$ and so such a vector would have to lie in the x-y plane. But then it couldn't be simultaneously orthogonal to $\mathbf{v}(\pi/2)$ and $\mathbf{v}(-\pi/2)$.

12. a) In the first case, $L = 20x + y - z - 9$. In the second, $L = 20x - y + 3z - 7$.
    b) In the first case, $L = z$. In the second, $L = 3y + z$.

13. a) In the first case, the plane is where $z = 1$. In the second, it is where $x + z = 1$.
b) In the first case, the plane is where $2x - y = 2$. In the second, it is where $x + y - z = 1$.

14. $L = 2x - 5y + z$.

15. a) 2. b) 1. c) $-\sqrt{2}$.

16. $\nabla g = (-2, -4)$.

17. a) $\nabla f = (-\sin(x), 2y)$ so possible the critical points have the form $(n\pi, 0)$ with n an integer. The second derivative tests finds $(n\pi, 0)$ a local minimum when n is odd and a saddle when n is even.
b) $\nabla f = (-\sin(x)\sin(y), \cos(x)\cos(y))$ so the critical points have the form $(n\pi, (m + 1/2)\pi)$ and $((n + 1/2)\pi, m\pi)$ where n and m are integers. In the first case, if both n and m are odd or both are even, it is a local maximum. If one is odd and the other not, then it is a local minimum. In the second case, it is a saddle for all n and m. (Use the 2nd derivative test.)
c) $\nabla f = (2x, 3y^2 - 3)$ so the critical points are $(0, \pm 1)$. The point $(0, 1)$ is a local minimum and the point $(0, -1)$ is a local minimum.

18. The origin is the only interior critical point and it is a saddle. The extreme points on the boundary occur at $(\pm 3/\sqrt{2}, \pm\sqrt{2})$. The points $(3/\sqrt{2}, \sqrt{2})$ and $(-3/\sqrt{2}, -\sqrt{2})$ are maxima and the others are minima.

19. The maxima occur at $(\pm 1, 0, 0)$ and the minima at $(0, \pm 1, 0)$.

20. The maximum is at $(1, 0, 0)$.

21. The closest points are $(0, \pm 1, \pm 1)$ with distance $\sqrt{2}$.

22. The square of the length of **E** is $x^4 + 4y^2 + z^2 + z^2y^2 - 2zy + 1$ which is smallest at the origin.

23. $3(\pi/2 - 1)$.

24. $4/(3\pi)$. (Remember to divide the integral of x by the area.)

25. $\pi/8$.

26. $\sin(1)/2$.

27. $\pi/2$

28. $\pi \sin(1)$.

29. $4\pi (1 - \cos(1))/3$.

30. $$\int_0^1 \left(\int_0^r \left(\int_0^\pi \left(r^3 \sin(\theta)\cos^2(\theta) \right) d\theta \right) dz \right) r\,dr = 1/9.$$

31. $$\int_0^{p/2} \int_0^{p/4} \left(\int_0^{1/\cos(\varphi)} \left(\rho^4 \sin(\theta)\cos(\theta)\sin^2(\phi)\cos^2(\phi) \right) \rho^2 d\rho \right) \sin(\phi)\, d\phi\, d\theta = 1/56.$$

32. $$\int_{-3/5}^{3/5} \int_{-\sqrt{9/25 - x^2}}^{\sqrt{9/25 - x^2}} \int_{\frac{4}{3}\sqrt{x^2 + y^2}}^{\sqrt{1 - x^2 - y^2}} dz\, dy\, dx =$$

$$\int_0^{4/5} \left(\int_{\frac{3}{4}r}^{\sqrt{1-r^2}} \left(\int_0^{2\pi} d\theta \right) dz \right) r\,dr =$$

$$\int_0^1 \left(\int_0^{\mathrm{Arc}\tan(4/3)} \left(\int_0^{2\pi} d\theta \right) \sin(\phi)\, d\phi \right) \rho^2\, d\rho = 4\pi/15.$$

33. a) $\int_0^\infty \int_0^{2\pi} e^{-r^2}\, d\theta\, r\,dr = \pi$.

b) The square of this integral is the Cartesian coordinate version of the preceding integral, so the integral in question equals $\pi^{1/2}$.

34. $u(t, x) = e^t (1 + (x - 4t)^2)^{-1}$.

35. a) Advection. b) Advection. c) Diffusion.

36. 0

37. $-2/\pi - 2/7$

38. 0.

39. a) $\mathbf{X}(u, v) = (2 (1 - u^2/9 - v^2/25)^{1/2}, u, v)$ for values of (u, v) with $u^2/9 + v^2/25 \le 1$.

b) $\mathbf{X}(u, v) = (u, 3 (1 - u^2/4 - v^2/25)^{1/2}, v)$ for values of (u, v) with $u^2/4 + v^2/25 \le 1$.

c) $\mathbf{X}(u, v) = (1 - u^2 - v^4, u, v)$ for values of (u, v) with $u^2 + v^4 \le 1$.

d) $\mathbf{X}(u, v) = (u, v, 2 - u - v)$ for values of (u, v) in $u \ge 0$, $v \ge 0$ and $u + v \le 2$.

40. a) $\int_{-5}^{5}\left(\int_{-3\sqrt{1-v^2/25}}^{3\sqrt{1-v^2/25}} \sqrt{1+4(u^2/81+v^2/625)/(1-u^2/9-v^2/25)}\,du\right)dv.$

b) $\int_{-5}^{5}\left(\int_{-2\sqrt{1-v^2/25}}^{2\sqrt{1-v^2/25}} \sqrt{1+9(u^2/16+v^2/625)/(1-u^2/4-v^2/25)}\,du\right)dv.$

c) $\int_{-1}^{1}\left(\int_{-\sqrt{1-v^4}}^{\sqrt{1-v^4}} \sqrt{1+4u^2+16v^6}\,du\right)dv.$

41. 10/3. (The area of the surface is π and the integral of z over the surface is $10\pi/3$.)

42. sin(1).

43. 500 $\pi/3$.

44. (x, -y, 0).

45. a) $(x^2yz/2, 0, 0)$.
b) (2z, 3x, y).

46. a) $(0, x/\pi)$.
b) (x, 0, 0).

47. a) No such vector field exists because (x, -2y, xy) has divergence -1 and the divergence of a curl is zero.
b) $(xy\cos(yz^2), 0, 0)$.

48. 5x + 3y = 0.

49. $\iint_S x\,g\,dS$. Let **E** = (g, 0, 0); here div(**E**) = f while **E•n** = x g on the surface of the volume, V, in question. Thus, the divergence theorem implies that $\iint_S x\,g\,dS = \iiint_V f\,dV$.

50. a) 2π . This is a direct computation: Parameterize the circle by $t \to (\cos(t), \sin(t))$ and then path integral is just the integral of dt between 0 and 2π.
b) 2π. Use Green's theorem for a region with holes and note that the vector field in question has zero 'curl' in the sense that when written as (P, Q), then $Q_x - P_y = 0$.
c) 0. This is also a direct application of Green's theorem.

51. A = (-1, 0, 0, 0, 3), B = (-1, -1, 0, 2, 2).

52. The mean is $m = (n_1 m_1 + n_2 m_2)/(n_1 + n_2)$ and the standard deviation is
$s = ((n_1 - 1) s_1^2 + (n_2 - 1) s_2^2)^{1/2}/(n_1 + n_2 - 1)^{1/2}$.

53. For 3.18: This is $Pr(B \cup C) = Pr(B) + Pr(C) - Pr(B \cap C) = .099$
For 3.19: This is $Pr(A \cup B \cup C) = .0143$.
For 3.20: This is $Pr(A \cap \overline{B} \cap \overline{C}) + Pr(\overline{A} \cap B \cap \overline{C}) + Pr(\overline{A} \cap \overline{B} \cap C) = .1368$
For 3.21: This is Pr(affected person is female) = .677.
For 3.22: This is Pr(both affecteds are female) = .263.
For 3.23: This is Pr(both < 80) = .160.

54. For 3.85: Use Baye's theorem to find $Pr(Y_1|(X_1 \cap X_6 \cap X_4)) = .009$.
For 3.87: You are computing $Pr(X_7|Y_2) = .7$.
For 3.88: Youi are computing $Pr(\overline{X}_7|\overline{Y}_2) = .605$.

55. For 3.104: .938.
For 3.105: .988.

56. For 4.37: Use the binomial distribution to find that the answer is .172.

57. For 4.44: The probability of the 82 year old dying in the next year is $p_1 = (l_{82} - l_{83})/l_{82} = .104$. Similar probabilities, $\{p_j\}_{2\le j\le 11}$ can be obtained for the others. The sum, $\sum_{1\le j\le 11} p_j = .176$, is the answer.

58. For 4.69: Use the binomial expansion with $n = 5$, $p = .4$ to find $Pr(X = 3) = .230$.
For 4.70: $Pr(X \ge 3) = .317$ using binomial table (Table 1) in Chapter 4.

59. For 4.78: Use the Poisson distribution with $\mu = 15.6$ to find the answer $\cong 7.651 \times 10^{-13}$.

60. For 5.31: This is given by $\Phi(-1.667) = 1 - \Phi(1.667) \cong .048$.
For 5.32: This is given by $\Phi(-3) = 1 - \Phi(3) \cong .0013$.

61. For 5.61: This is $84!/(29! \times 55!)\,(.24)^{29}\,(.76)^{55} \cong .009$
For 5.62: Use the fact that $Pr(X \ge 29) \cong Pr(Y \ge 28.5)$ where Y is normally distributed with mean = np = 20.16 and variance = npq = 15.32. Thus, $Pr(Y \ge 28.5) \cong .017$.

62. For 5.64: $Pr(X \ge 90) = 1 - \Phi(2.307) = .0105$.
For 5.65: Approximate the binomial distribution with a normal one of mean np = 21.1 and variance npq = 20.8. Then, $Pr(X \ge 25) \cong 1 - \Phi(.0755) = .225$.