Math 21a Practice Exam 1 - (from Fall 1999)

This was a 90 minute exam.
Problems 1-6 counted 16% each, while Problem 7 counted 4%. Problem 6 was presumed to be quite difficult.

1. Let **v** = (2, 1, 4) and **w** = $\frac{1}{3}$(1, 2, 2).

a) Find the lengths of **v**, **w** and **v - 3w**.
b) Find the scalar projection of **v** in the direction of **w**.
c) Find **v** $\times$ **w**.
d) Find numerical values for the constants b and c so that both **v** and **w** are tangent to the plane where bx + cy + z = 0.

2. Let P denote the plane where 2x - 2y + z = 3.

a) Find three points in P which do not all lie on the same line.
b) Find a non-zero vector which is perpendicular to P.
c) Find the distance from P to the origin.
d) Write an equation for a line which lies entirely in P.

3. A particle moving in space has position at time t given by (3 sin(t^2), 3 cos(t^2), 4 t^2).

a) Find the coordinates of the particle at t = $\sqrt{\pi}$.
b) Find the velocity vector of the particle at t = $\sqrt{\pi}$.
c) Find a parametric equation for the line tangent to the trajectory at t = $\sqrt{\pi}$.
d) Find the distance traveled by the particle between t = 0 and t = $\sqrt{\pi}$.

4. Let **v** = (5, 3, 1) and **p** = (1, 0, 1). It turns out that the end points of the vectors **r** which obey (**r** - **p**) $\times$ **v** = 0 all lie on the same line, L.

a) Find a point on L and a tangent vector to L.
b) Write a parametric equation for L.
c) Find a point on L with distance 5 from the origin.
d) Find the closest distance from L to the origin.

5. Let **v** = (5, 3, 1) and let L denote the line through the origin with tangent vector **v**.

a) Find an equation for some plane through the origin which <u>contains</u> L.
b) Let **w** = (-10, b, 2). Find a value for b which makes **w** perpendicular to **v**.
c) Let **r** = (-10, b, c). Find values for b and c which makes **v** and **r** parallel.
d) Suppose that **u** = (1, 3, -2) and **s** = (-30, -18, c). Find a value for c which makes **u** $\times$ **s** perpendicular to **v**.

6. The position of a particle in space at time t is the head of a vector **r**(t) based at the origin. Use **k** to denote the vector (0, 0, 1) and suppose that **r**(t) - **k** (**k**•**r**(t)) is orthogonal at all times to the vector **r**´(t) - **k** (**k**•**r**´(t)). Also, suppose that the particle does not move along any one fixed line in space. Lable each of the statements below with

- 'A' if it is true for any and all **r**(t) as described above;
- 'B' if it is true for some, but not every **r**(t) as described above;
- 'C' if it is never true for any **r**(t) as described above.

a) The particle moves on the surface of a sphere centered at the origin.
b) The particle moves on the surface of a plane.
c) The particle moves on the surface of a cylinder.
d) The y coordinate of the particle is constant.

Please explain your reasoning in a sentence of two.

7. Let **u** = (3927, 42, -999) and **v** = (195, 735, 1115). What is (**u**/3 - **v**/5)•(**u** × **v**)?

Math 21a Hourly 1 Answers

(Fall 1999)

Problem 1:
a) |**v**| = $\sqrt{21}$, |**w**| = 1, |**v** - 3**w**| = $\sqrt{6}$.
b) This number is **v**•**w** = 4.
c) (-2, 0, 1).
d) b = -2 and c = 0.

Problem 2:
a) (3/2, 0, 0), (0, -3/2, 0) and (0, 0, 3). (There are infinitely many other possibilities)
b) (2, -2, 1), or any non-zero multiple
c) The distance is 1.
d) Parametric: Send t → (3t, 3t, 3). Nonparametric: x = y & z = 3 - 2x + 2y.
(There are infinitely many other possibilities.)

Problem 3:
a) (0, -3, 4π).
b) $2\sqrt{\pi}$ (-3, 0, 4)
c) t → (-3t, -3, 4 (π + t)).
d) The velocity vector at general t is 2t (3 cos(t^2), -3 sin(t^2), 4) whose length is 10 t.
Thus, the distance is the integral of this last function from 0 to $\sqrt{\pi}$ which is $5\sqrt{\pi}$.

Problem 4:

a) **p** is on L and **v** is tangent to L.

b) t $\rightarrow$ **p** + t**v**.

c) (-4, -3, 0); the case of t = -1 in the preceding parameterization.

d) d = |**p** $\times$ **v**|/|**v**| = $\sqrt{\frac{34}{35}}$.

Problem 5:

a) The plane where x - 3z = 0. (There are infinitely many other possibilities.)

b) **w•v** = 0 requires b = 16.

c) b = -6 and c = -2.

d) If **s** is parallel to **v**, then **u** $\times$ **s** will be perpendicular to **v**. For this, take c = -6.

Problem 6:

a) B, for you can take **r**(t) = (cos(t), sin(t), 0). This lies on the surface of the radius 1 sphere and satisfies the conditions. But, **r**(t) = (cos(t), sin(t), t) also satisfies the condition, but traces out a corkscrew path.

b) B, take the same **r**(t)'s as in the answer for 6a for justification.

c) A, indeed, write **r**(t) = (x(t), y(t), z(t)) and then **r** - **k** (**k•r**) = (x(t), y(t), 0). Likewise, **r**´ - **k** (**k•r**) = (x´, y´, 0), so our condition says that the motion in the x-y plane is just along a circle of fixed radius. Add in the z-coordinate (about which we know nothing) and we see that the motion is along a cylinder stretching off along the z-axis.

d) C, if the motion in the x-y plane is along a circle, and the particle does not have constant x-y coordinate (which would mean its total motion was along a line parallel to the z-axis), then the y-coordinate must change with time since the y coordinate is not constant on a circle.

Problem 7:

0. The vector **u** $\times$ **v** is perpendicular to both **u** and **v**.